Derivations Systems of Transparent Intensional Logic

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Logika: systémový rámec rozvoje oboru v ČR a koncepce logických propedeutik pro mezioborová studia (reg. č. CZ.1.07/2.2.00/28.0216, OPVK)

doc. PhDr. Jiří Raclavský, Ph.D. (*raclavsky@phil.muni.cz*)

Department of Philosophy, Masaryk University, Brno

Abstract

Materna's explication of the notion of conceptual system in Transparent Intensional Logic is insufficient for explication of our conceptual scheme even after improving his proposal by several ways. We have not only concepts at our disposal, we do reason with concepts. The entities consisting in rules operating on the domain of concepts will be called derivation systems. In formulation of the notion of derivation system we employ Tichý's system of deduction. Derivation systems differ from conceptual systems especially in including derivation rules. This enables us to show close connections among the realms of objects, their concepts, and reasoning with concepts. Derivations systems thus differ from conceptual systems as Peano's arithmetic from class of natural numbers.

Content

I.

Concepts – from extensional to hyperintensional conceptions

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I. Logical theory of concepts (a selective history)

- classical tradition – concept is general, extension/intension of concept, etc.
- Bolzano (Wissenschaftslehre) – concepts as abstract (non-psychological) entities, concepts need not to be general, structure of compound concepts
- Frege (Funktion und Begriff) – concept is an abstract entity, concept is predicative (i.e. general), falling under concept, concepts modelled as (Frege's) functions?
- Church (Introduction to Math. Logic) – generalizing Frege's conception (more below), concept need not to be general
- modern tradition – concepts modelled by means of set-theoretical entities
- Bealer (Quality and Concept) – concepts modelled as (Bealer's) intensions
- late Materna (Concepts and Objects) – concepts modelled as Tichý's hyperintensions

I. From extensional to intensional theory of concepts

- a common philosophical construal:
- an expression expresses a concept of a property (which has an extension); e.g. "man" expresses MAN, which *determines* the property *BE A MAN*, having an extension such as {Alan, Bill, ...}
- in *extensional* set-theoretical conception of concept (classical first-order logic), concept as well as property as well as extension of a property is explicated as a set, which is a very strong reduction
- inadequacy: lack of distinguishing between *empirical*, e.g. MAN, an *non-empirical*, e.g. PRIMES, *concepts*, i.e. ignorance of modal (and temporal) variability (while the extension of a non-empirical concept is modally stable, the extension of an empirical concept varies)

I. From intensional to hyperintensional theory of concepts

- intensional logic offers tools for modelling of modal (and temporal) variability viz. intensions
- *intensions* are set-theoretical objects – they are functions from possible worlds (and time-moments)
- thus intensional logic can model *property* as distinct from its extensions, namely as an intension having classes of objects (i.e. extensions of the property) as their values
- possible modelling of concepts by means of intensional logic
- success of intensional theory of meaning (propositional attitudes, intensional transitives, etc.) in 1970's; its failures (paradox of omniscience) recognized mainly in 1980's; the quest for hyperintensional entities

I. Towards hyperintensional theory of concepts

- Bolzano's lesson: a concept does not have a set-theoretical structure, it has a structure finer than a set (sum) of its parts
- Bolzano's example (evoked by Pavel Materna): A LEARNED SON OF AN UNLEARNED FATHER vs. AN UNLEARNED SON OF A LEARNED FATHER; the two concepts have the same content {UN-, LEARNED, SON, FATHER}
- mathematical examples: THREE DIVIDED BY TWO vs. TWO DIVIDED BY THREE (content={2,2,÷}), or $1\times2=3-1$ vs. $1=3-(2\times1)$ (content={1,2,3,-,×})
- (what is symptomatic of a composition of a compound concept, i.e. of its complexity?)

I. Church's theory of concepts

- semantic scheme (Introduction to Mathematical Logic, ...)

an expression

| *expresses*

concept (*sense*)

| *determines* (an expression *denotes*)

object (*denotation*)

- lack of intensional variability in the modern sense, i.e. not distinguishing between empirical and non-empirical concepts
- modelling concepts; a concept of an individual is a member of ι_1, (an individual is a member of ι_0), a concept of such concept is a member of ι_2 (analogously up); this means that concepts are modelled in a trivial way (i.e. their internal structure is wholly neglected)

II.

Tichý's logical framework

II. Tichý's logical framework: Transparent intensional logic and meaning

- from 1971 (see Tichý 2004, Collected Papers)
- constructions are abstract, hyperintensional entities, procedures (more below)
- semantic scheme:

an expression *E*

| *expresses* (*means*) in *L*

a construction (i.e. the *meaning of E* in *L*)

| *constructs*

an intension / non-intension / nothing (*cf.* "3÷0")

(i.e. the *denotatum* of *E* in *L*)

- the value of an intension in possible world *W* at time-moment *T* is the *referent* of an empirical expression *E* in *L* (the denotatum and referent of a non-empirical expression are identical)

II. Tichý's logical framework: on the nature of constructions

- the very same function (as mapping):

$$\begin{array}{ccc} \vdots & & \vdots \\ 1 & \rightarrow & -2 \\ 2 & \rightarrow & 1 \\ 3 & \rightarrow & 6 \\ \vdots & & \vdots \end{array}$$

can be reached by many different – albeit equivalent – *procedures*

written as "'$\lambda x[[x{\times}x]-3]$", "$\lambda x\, [[x + x^2] - [3 + x]]$", etc.

- Tichý's *constructions*, exactly defined in Tichý 1976-1988, are such procedures
- they are akin to algorithms
- one may view them as objectual correlates of lambda terms
- they are extralinguistic entities; they are not expressions (not even λ-terms!)
- they are not set-theoretical entities; they are not classes or any other functions

II. Tichý's logical framework: Simplified definition of constructions

1) a *variable* x^k is a construction which constructs, dependently on valuation v, the entity which is the k-th member of the sequence of entities which is v

2) if X is an object or construction, *trivialization* of X, ${}^0X$, is a construction which constructs X (${}^0X$ takes X and leave it as it is)

3) if C, C_1, ..., C_n are constructions, then their *composition* $[C\ C_1...C_n]$ (v-)constructs the value (if any) of the function constructed by C on the respective argument constructed by C_1, ..., C_n; otherwise it (v-)constructs nothing (is v-improper)

4) a variable x occurring in C can be abstracted upon; the construction λxC is an *x-closure of* C and it constructs a function from possible values of x to entities constructed (on the respective valuations for x) by C

- thus *constructions are specified as ways of constructing objects*

II. Tichý's logical framework: deduction

- Tichý's pioneering and excellent work on the deduction in partial type theory (1982, 1986; originally 1976)
- the deduction system is a system of *sequents*, which is, again, fully objectual, i.e. not amenable to model-theoretic (or other) interpretation
- the basic entities are so-called *matches* X:C where X is a (trivialization) of an object v-constructed by C or a variable ranging over such objects; sequents are made from matches and *derivation rules* are made from sequents
- derivation rules are made from constructions and objects, which leads to the fact that *derivation rules display properties of objects* (JR+PK 2011)

III.

Materna's Concepts

III. Materna's theories of concept (1.)

- *Pavel Materna*'s proposal (1989) according to which (empirical) concepts are intensions was criticized (in a review of his book) by *Pavel Cmorej* who suggested utilizing Tichý's notion of construction
- next proposal by Materna (after 1989, til 1998): concept is a class of α- and η- (or pre-1998: γ-) convertible closed constructions
- *Aleš Horák* (2002): *concept* is a closed construction is a α- and η-normal form; this was adopted by Materna (2004)
- after 2004 (with Duží, Jespersen): various attempt to define a convenient *procedural isomorphism*, i.e. congruency of concepts imagined by Church in his various Alternatives

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III. Materna's theories of concept (2.)

- Materna adopted Church's 1956 scheme: expressions expresses concepts, etc.
- being entirely so, Materna's concepts would be the same as Tichý's constructions; the demand of closeness and α- and η-normal forms is nothing but the way how to differentiate concepts from constructions, i.e. concepts from meanings
- some important philosophical applications are developed by Materna (e.g. emptiness of concepts), some are still missing (e.g. a Frege-like falling under concept)

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III. Materna's theories of concept (3.)

- some details of Materna's 1998 key idea
- a *concept* is a Tichý's construction which is:

i) *closed*

because, e.g., $[{}^0\mathrm{Man}_{wt}\, x]$ is hardly a concept, only the closed construction ^{0}Man is one

ii) *in α- and η-* (but not β-) *normal form*

because the difference between α- (e.g. $\lambda n\, [n\ {}^0{>}\ {}^0 7]$ vs. $\lambda m\, [m\ {}^0{>}\ {}^0 7]$) or η-equivalent (e.g. $\lambda w \lambda t\, [\lambda x\, [{}^0\mathrm{Man}_{wt}\, x]]$ vs. ^{0}Man) constructions is minimal and perhaps somehow artificial

- since concepts are constructions, *concepts are abstract structured procedures*; most expressions express concepts

III. Materna's theories of concept (4.)

- most Materna's applications are in principle acceptable to us, e.g.:
- *simple* / *compound* concepts (Materna: primitive/ derived) – due to the definition of subconstructions
- concepts are *equivalent* iff they construct one and the same object
- a concept *C* is *empirical* iff *C* determines (i.e. constructs) an intension (*C* is *non-empirical* otherwise)
- *emptiness* of concepts – more versions (strict: THREE DIVIDED BY ZERO, 'normal': ROUND SQUARE, empirical: UNICORNS, PEGASUS, THE KING OF FRANCE)
- an *intension* of a concept *C* = a class of simple subconcepts of *C*
- an *extension* of *C* (in *W*,*T*) = a class of objects falling (in *W*,*T*) under *C* (i.e. the value of the function constructed by *C*)

IV.

Materna's conceptual systems

IV. Materna's conceptual systems (1.)

- simple version in Materna (1998)
- *conceptual system* is

$$CS = SC_{CS} \cup CC_{CS}\text{, whereas}$$

- SC_{CS} (Materna: "primitive concepts") is a class of some 1st order *simple* concepts (i.e. trivializations), and
- CC_{CS} (Materna: "derived concepts") is a class of *all compound* concepts made from members of SC_{CS} and variables by ways of forming constructions (e.g. by trivializing)
- any CS is uniquely given just by SC_{CS}

IV. Materna's conceptual systems (2.)

- Materna 1998 (with a slight modification)
- an object *O* is *dealt by CS* iff at least one concept of *CS* determines *O*
- a class *S* is the *area of CS* iff *S* is the class of all concepts dealt by *CS*
- classification of conceptual systems: *empirical*/*non-empirical systems* (an empirical system deals at least one non-trivial intension), etc.

- comparing conceptual systems w.r.t. their areas (in more details in Materna 2004): weaker than/stronger than, equivalent to, being a part of, independent of, being an (possibly: essential) extension-expansion of, etc.
- many Materna's results are acceptable for us

IV. Materna's conceptual systems (3.)

- critical remarks on Materna's theory of conceptual systems:
- a. for each *CS*, there is just 1 relation operating on ($SC_{CS} \cup CC_{CS}$) (i.e. the *concept domain* of that *CS*), namely *BEING BUILT ONLY FROM* ...; yet even more relations operating on the concept domain are thinkable
- b. each *CS* has a concept domain of the same cardinality – which is counterintuitive
- c. the cardinality of every *CS* is infinite – which seems also counter-intuitive
- d. each *CS* contains concepts of concepts (these *conceptualized* conceptual systems can be seen as only some of conceptual system there are)
- e. moreover, each *CS* contains an infinite number of definitions which is counter-intuitive; (according to Materna, every compound concept is a definition of the object determined by it)
- f. some problems related to CSs cannot be even formulated in Materna's theory

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IV. Materna's conceptual systems (4.)

- JR+PK (2011): CC is in fact a class of compound concepts, not derived concepts (as Materna suggests); SC is class of simple concepts, not primitive concepts (as Materna suggests)
- thus JR+PK (2011): especially the CC-part can be changed and the notion of CS could be even more fruitful; for example:

CS=<{primitive concepts}, {ordering relations}>

(JR+PK 2010 in Toruń, not in our 2011-paper where it was omitted)

- possible ordering relations – being a simple (alternatively: compound) concept, being a subconcept of, having only C_1, ..., C_n as its subconcepts, being empirical, being definable (in this or that derivable system) by means of, etc.
- the same modification is welcome also for Materna's second proposal

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IV. Conceptual Systems: Materna's second proposal (3.)

- Materna's more complicated explication (2004)
- it utilizes Tichý's 1988 idea that a system of construction is only defined over a well-defined system of objects; to illustrate (JR+PK 2011) two such systems can differ significantly; imagine arithmetic without or with the number 0, whereas numbers are in an atomic type: the function ÷ is or is not partial, thus [3÷0] is an abortive procedure of only the first system

CS = < <1. atomic types, 2.Tichý's definition of type theory>,
<3.{SCs over atomic types}, 4.{modes of forming constructions},
(arbitrary?) 5. definitions of α- and η-normalization> >

- JR+PK (2011): the part 4. {modes of forming constructions} is redundant because it is covered already by Tichý's definition of type theory

V.

Derivation systems

V. Derivation systems: the basic idea

- the greatest disadvantage of Materna's TIL is a complete ignorance of Tichý's deduction
- Materna's conceptual systems are only fields for deduction
- though it is certainly true that we have concepts collected in conceptual systems, it is also hardly deniable that we *ratiocinate* with these concepts; framed within a conceptual system, we perform operations - including inferences - with concepts, exploiting various *derivation rules*
- from 2009, I call the entities involving both concepts and rules *derivation systems*
- derivation systems differ from conceptual systems the same way as Peano's arithmetic from the quite uninteresting set {1,2, ..., +, Succ, ...}

V. Derivation systems: the simple proposal

- a simple definition useful for many purposes (already JR 2007):

DS=<C_{DS}, R_{DS}>,

where C_{DS} is a class of constructions (not only concepts; the aim: a generality and thus also fruitfulness of the notion) and R_{DS} a class of derivation rules operating on C_{DS}

- (Materna's CSs are at best certain DSs having 0 derivation rules)

- key idea: *derivation systems DSs are tools for stating and proving facts about objects which are constructed by members of* C_{DS}

V. Derivation systems: the definition

- jointly with P. Kuchyňka, who developed the notion independently, a precization was made
- *derivation system DS* is a quintuple:

$<OB_{DS}$, defTT, PC_{DS}, Q_{DS}, $R_{DS}>$,

where:

– OB_{DS} is a particular *objectual basis* (aka 'atomic types', but see Tichý)
– *defTT* is the definition of Tichý's theory of types
– PC_{DS} ('*primitive concepts*') is a particular class of trivializations, a subclass of the class *AC* of all constructions over OB_{DS}

V. Derivation systems: the definition (cont.)

DS=<OB_{DS}, defTT, PC_{DS}, Q_{DS}, R_{DS}>

– Q_{DS} is a particular class of *qualities* of constructions from *AC* (i.e. properties such as *HAVING THE ORDER K*, *HAVING C AS ITS SUBCONSTRUCTION*, *HAVING THE COMPLEXITY-RANK R*), the 'conjunction' of all these qualities characterizes the class CR_{DS} (= non-primitive constructions occurring in rules) which is that subclass of *AC* which contains *all* constructions occurring in members of R_{DS}

– R_{DS} is a particular class of derivation *rules* whereas sequents involved in these rules are made from matches of form $X{:}C$ where X is a variable or a member of PC_{DS} and C is a member of CR_{DS}

VI.

Concluding remarks

VI. Concluding remarks: general impact of derivation systems on for the study of concepts and conceptual systems

- in DSs, implicit relations between constructions (or objects) are made explicit, which is inevitable if we need to move from our intuitive use of concepts towards their conscious reflection
- DSs are thus important for the study of concepts (and other constructions) especially from the *methodological point of view*, because they enable us to *precisely specify conceptual systems* and to *prove claims about them*
- they yield rigorous and controllable results, ‘nothing is left to guessing’ (Frege)
- for applications of derivations systems see my slides “Interaction of Semantics and Deduction in Transparent Intensional Logic” and “Derivation Systems and Verisimilitude (An Application of Transparent Intensional Logic)”

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