Solution to Semantic Paradoxes in Transparent Intensional Logic









INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Logika: systémový rámec rozvoje oboru v ČR a koncepce logických propedeutik pro mezioborová studia (reg. č. CZ.1.07/2.2.00/28.0216, OPVK)

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Abstract

We propose a solution to semantic paradoxes pioneered by Pavel Tichý and further developed by the present author. Its main feature is an examination (and then refutation) of the hidden premise of paradoxes that the paradox-producing expression really means what it seems to mean. Semantic concepts are explicated as relative to language, thus also language is explicated. The so-called 'explicit approach' easily treats paradoxes in which language is explicitly referred to. The residual paradoxes are solved by the 'implicit approach' which employs ideas made explicit by the former one.

I. Introduction: semantic paradoxes

- semantic paradoxes (SPs) e.g., Liar, Berry's p., Grelling's heterological p. ...
- the paradox-producing expression always includes some semantic term such as "true", "denote", "refer"
- last 100 years: more than 900 papers and books on SPs (90% about Liar) and semantic terms (90% about truth-predicate)
- last decade: increasing interest in the paradoxes of denotation and reference (e.g., Simmons 2003, Priest 2006, Field 2008)

I. Introduction: solutions to SPs

- solutions to SPs have to detect what is wrong with
 - a. our naïve theory of semantic terms, or
 - b. our ordinary, naïve inference rules and suggest a plausible *critical theory*, replacing thus a. or b.
- classical (hierarchical) approaches by Russell and Tarski, three-(and more)valued approaches by Łukasiewicz, Kripke, etc.
- recent domination of rather non-classical approaches: paraconsistent logic (dialetheias, Priest), revision theory (circular concepts and definitions, Gupta & Belnap), paracompleteness (roughly: non-standard rules, Field), contextualism (e.g., Simmons)

I. Introduction: Transparent Intensional Logic (TIL)

- logical theory developed by Pavel Tichý from early 1970s
- his semantic doctrine, i.e. (logical) explication of meanings, has many successful applications (see esp. Tichý 2004 collected papers, Tichý 1988, recently Duží & Jespersen & Materna 2010)
- TIL is capable to solve also SPs of denotation and reference
- the solution here presented is inspired by Tichý's solution to Liar (1976, 1988), there several writings by the present author (2009-2011) solving all known paradoxes of denotation and reference

I. Introduction: about the TIL-approach to SPs

- critical examination (and then refutation) of the *hidden premise* of SPs that the paradox-producing expression *means* what it seems to mean (generalized from Tichý 1988)
- semantic concepts are explicated as inescapably relative to language (mostly in Raclavský 2009) thus also the concept of language is explicated (ibid.)
- recourse to fundamental truism that an expression *E* may mean / denote / refer to something only relative to a particular language

Content

- II. TIL-basics, i.e. constructions, deduction, explication of meanings (semantic scheme), type theory
- III. 'Explicit approach', i.e. explication of language, explication of semantic concepts as *explicitly* relative to a language, solution to SPs
- IV. 'Implicit approach', i.e. an objection the revenge problem, semantic concepts which are *implicitly* relative to a language, solution to residual SPs
- V. Conclusions

II. TIL basics

- objects, functions and constructions
 - deduction
 - type theory

II. TIL basic: functions and constructions

- two notions of function (historically):
 - a. as a mere mapping ('graph'), i.e. function in 'extensional sense',
 - b. as a structured recipe, procedure, i.e. function in 'intensional sense'
- Tichý treats functions in both sense:
 - a. under the name functions,
 - b. under the name constructions
- an extensive defence of the notion of construction in Tichý 1988

II. TIL basics: objects and their constructions

- constructions are structured abstract, extra-linguistic procedures
- any object *O* is constructible by infinitely many *equivalent* (more precisely *v-congruent*, where v is valuation), yet *not identical*, constructions (='intensional' criteria of individuation)
- each construction *C* is specified by two features:
 - i. which object O (if any) is constructed by C
 - ii. how C constructs O (by means of which subconstructions)
- note that constructions are closely connected with objects

II. TIL basics: kinds of constructions

- five (basic) kinds of constructions (where X is any object or construction and C_i is any construction; for exact specification of constructions see Tichý 1988):

a. variables	χ	('variables')
b. trivializations	^{0}X	('constants')
c. compositions	$[C C_1C_n]$	('applications')
d. closures	λχC	('λ-abstractions')
e. double executions	^{2}C	(it v -constructs what is v -constructed by C)

- definitions of subconstructions, free/bound variables ...
- constructions *v*-constructing nothing (c. or e.) are *v*-improper
- recall that constructions are not formal expressions; λ -terms are used only to denote constructions which are primary

II. TIL basics: deduction and definitions

- Tichý's papers on deduction (though only within STT) in 2004
- because of partiality, classical derivation rules are a bit modified (but not given up)
- match X:C where X is a (trivialization of O), variable for Os or nothing and C is a (typically compound) construction of O
- *sequents* are made from matches
- derivation rules are made from sequents
- note that derivation rules *exhibit properties of* (and relations between) *objects and their constructions* (Raclavský & Kuchyňka 2011)
- viewing definitions as certain \Leftrightarrow -rules (*ibid.*); explications

II. TIL basics: simply type theory (STT)

- (already in Tichý 1976; generalized from Church 1940): let *B* (basis) be a set of pairwise disjoint collections of objects:
 - a. every member of B is an (atomic) type over B
 - b. if ξ , ξ_1 , ..., ξ_n are types over B, then $(\xi \xi_1 ... \xi_n)$, i.e. collection of total and partial functions from $\xi_1,...,\xi_n$ to ξ , is a (molecular) type over B
- for the analysis of natural discourse let B_{TIL} = {1,0, ω , τ }, where 1 are individuals, 0 are truth-values (T and F), ω are possible worlds (as modal indices), τ are real numbers (as temporal indices)
- functions from ω and τ are intensions (propositions, properties, relations-in-intension, individual offices, ...)

II. TIL basics: semantic scheme

- somewhat Tichýan semantic scheme (middle 1970s):

an expression E "The Pope is popular"

| expresses (means) in L |
a construction = the meaning of E in L | $\lambda w \lambda t \, [^{0}Popular_{wt}\, ^{0}Pope_{wt}]$ | constructs |
an intension / non-intension | that proposition (i.e. $\langle W_{1}, T_{1} \rangle \to T$,
= the denotatum of E in L | $\langle W_{1}, T_{2} \rangle \to F$, $\langle W_{1}, T_{3} \rangle \to (i.e. gap)$, ...

- the value of an intension in possible world *W* at moment of time *T* is the *referent* of an *empirical expression E* ("the Pope", "tiger", "It rains") in *L*; the denotatum and referent (in *W* at *T*) of a *non-empirical expression E* ("not", "if, then", "3") are identical

II. TIL basics: hyperintensionality

- intensional and sentencialistic analyses of belief sentences are wrong
- Tichý: belief attitudes are attitudes towards constructions of propositions (not to mere propositions or expressions); i.e. the agent believes just the construction expressed by the embedded sentence
- e.g., "Xenia believes that the Pope is popular" expresses the 2nd-order construction:

λwλt [ºBelieve_{wt} ºXenia ºλwλt [ºPopular_{wt} ºPope_{wt}]]

(note the role of the trivialization; $(oi^*_1)_{\tau\omega}$); another example: "X calculates $3\div 0$ " expresses $\lambda w \lambda t \, [^0\text{Calculate}_{wt}\, ^0\text{X}\, ^0[^03^0\div ^00]]$

II. TIL basics: ramification of type theory (TTT)

- treatment of constructions *inside* the framework necessitates the ramification of the type theory
- cf. precise definition of TTT in Tichý 1988, chap. 5
 - 1. STT given above = first-order objects
 - 2. first-(second-, ..., n-)order constructions (members of types $*_1$, $*_2$, ..., $*_n$)
 - = constructions of first-(second-, ..., n) order objects (or 2nd-, ..., n-1-order constructions)
 - 3. functions from or to constructions
- (Church like) cumulativity: every k-order construction is also a k+1-order construction
- 'speaking about types' in TTT with a basis richer than B_{TIL}

II. TIL basics: four Vicious Circle Principles (VCPs)

- understanding TTT as implementing four Vicious Circle Principles (VCPs), e.g.
 Raclavský 2009
- each of them is a result of the *Principle of Specification*: one cannot specify an item by means of the item itself
- Functional VCP: no function can contain itself among its own arguments or values (cf. STT, 1.)
- Constructional VCP: no construction can (v-)construct itself e.g., a variable c for constructions cannot be in its own range, it cannot v-construct itself (cf. RTT, 2.)
- Functional-Constructional VCP: no function F can contain a construction of F among its own arguments or values (cf.3)
- *Constructional-Functional VCP*: no construction *C* can construct a function having *C* among its own arguments or values (*cf.* 2. and 3.)

II. TIL basics: some conclusions about the approach

- unlike rivalling solutions to SPs (cf. Gupta & Belnap, Field, Priest, ...):
- a. it is explicitly stated what meanings are (meanings are constructions)
- b. this semantical theory is hyperintensional (not intensional or extensional), i.e. its underlying TT is ramified
- c. the system is rather classical: bivalency and classical logical laws are accepted, yet it treats partiality (thus logical laws are a bit corrected)
- d. the approach is rather general, it treats many logical phenomena (it is not a logic designed to a single, particular problem)
- e. the overall feature is its objectual (not formalistic) spirit

III. 'Explicit approach'

- 1. language as a hierarchy of codes
- 2. explication of semantic concepts
- 2. explicit solution to semantic concepts

III. Explicit approach: hierarchy of codes (1.)

- language is (normative) system enabling speakers to communicate
- restricting rather to the model of its coding means, i.e. language is *explicated* as function from expressions to meanings
- a k-order code L^k is a function from real numbers (incl. Gödelian numbers of expressions) to k-order constructions, it is an $(*_k\tau)$ -object (Tichý 1988)
- there are various 1st-, 2nd-, ..., n-order codes (Tichý 1988)

III. Explicit approach: hierarchy of codes (2.)

- it is not sufficient, however, to model coding means of (say) English by a single (say 1st-order) code
- rather, whole hierarchy of codes should be invoked as a model of English
- key reason: English is capable to code (express by some its expression) constructions of higher orders

(recall that, e.g. [... c^1 ...], where c^1 is a variable for 1st-order constructions, is a 1+1-order construction)

- for the philosophical justification and details concerning reductive nature of the model of language see (JR 2014a)

III. Explicit approach: hierarchy of codes (3.)

- any construction of L^1 , most notably ${}^0L^1$, is among constructions not expressible in L^1 (recall Functional-Constructional VCP: if ${}^0L^1$ would be a value of L^1 , which is a function, L^1 were not be specifiable)
- ${}^{0}L^{1}$ is the meaning of the name of L^{1} , i.e. " L^{1} " (we need ${}^{0}L^{1}$ for the explication of the meaning of "... in L...")
- remember:
 - a. no construction of a k-order code L^k is codable in L^k

(only in a higher-order code)

b. no expression mentioning (precisely: referring to) L^k is endowed with meaning in L^k (only in a higher-order code)

III. Explicit approach: hierarchy of codes (4.)

- a hierarchy of codes involves (= conditions on hierarchies):
 - 1. n codes of n mutually distinct orders ($L^1, L^2, ..., L^n, ...$)
 - 2. each expression E having a meaning M in L^k has the same meaning M in L^{k+m}
 - 3. an expression *E* lacking meaning in L^k can be meaningful in L^{k+m}
- of course, most of the everyday communication takes place in the 1st-order code L^1 of the hierarchy
- higher-order coding means (e.g., L^2) are invoked rarely only when one comments parts of English by means of another part of English (my implementation of the universality-of-language principle)

III. Explicit approach: hierarchy of codes (technical remarks)

- every code of the same hierarchy shares the same expressions, quantification over all of them is unrestricted
- due to order-cumulativity of functions, every k-order code is also a k+1-order code; thus the type $\binom*{}_n\tau$ includes (practically) all codes of the hierarchy; we can quantify over them
- a hierarchy of codes is a certain class (it is an $(o(*_n\tau))$ -object); one can quantify even over families
- note that a hierarchy of codes is a 'system' of coding vehicles, not a particular vehicle ('language'); thus we investigate meanings of expressions in members of the hierarchy, e.g. L^n , not in the hierarchy as a whole

III.2 Explicit approach: explication of semantic concepts (1.)

- sample definitions (e.g. Raclavský 2009; definitions can be viewed as explications of the intuitive concepts):

- $[l^n n]$ *v*-constructs the value of an *n*-order code L^n for the expression N, i.e. N's meaning in L^n
- note the simplicity and material adequacy of the model

III.2 Explicit approach: explication of semantic concepts (2.)

- truth as a *property of propositions* (in the range of *p*):

$$[{}^{0}\text{True}^{\pi P}_{wt} p] \iff p_{wt}$$

(partial sense: a proposition P can be neither true^{πP} or false^{πP})

$$[{}^{0}\text{True}^{\pi T}_{wt} p] \Leftrightarrow {}^{0}[{}^{0}\exists \lambda o [[o {}^{0}=p_{wt}] {}^{0} \land [o {}^{0}={}^{0}T]]]$$

(total sense: a proposition P is true^{πT} or not true^{πT}; an analogue is in Tichý 1988)

- truth as a property of constructions (4 kinds; again, Raclavský 2008); a construction is $true^{*n}$ in w, t iff it v-constructs a proposition $true^{\pi}$ in w, t

III.2 Explicit approach: explication of semantic concepts (3.)

 truth as a property of expressions (language relative, 6 principal kinds, Raclavský 2009)

[°TrueIn
$$^{P}_{wt} n l^{n}$$
] \Leftrightarrow [°True $^{\pi P}_{wt} {}^{2}[l^{n} n]$]

[°TrueIn $^{T}_{wt} n l^{n}$] \Leftrightarrow [° $\exists \lambda o [[o {}^{0} = {}^{2}[l^{n} n]]_{wt}] {}^{0} \wedge [o {}^{0} = {}^{0}T]]$]

- note the relation of truth to other semantic concepts: an expression N is true in L^n iff it *expresses-means* in L^n a construction of a true^{π} proposition, i.e. it *denotes* in L^n a true^{π} proposition, i.e. it *refers* L^n to T

III.3 Explicit approach: solution to particular SPs (1.)

- D: "1 + the denotatum of D" (the Paradox of Adder)
- a. if we do really understand D, we are capable to single out in which language-code the denotation should proceed; i.e. we thus *disambiguate* D to (say) "1 + the denotatum of D in L^1 " (hereafter simply D)
- b. we thus understand D by means of (say) the 2nd-order code L^2 (or L^3 , ...) of English
- c. in L^2 , D means the 2nd-order construction [01 $^0+$ [$^0TheDenotatumOfIn^{\tau}$ $^0g(D)$ $^0L^1$]]
- d. being a 2nd-order construction, it cannot be expressed by D already in the 1st-order code L^1 ; thus D is meaningless in L^1

III.3 Explicit approach: solution to particular SPs (2.)

- e. lacking meaning in L^1 , D has no denotatum in L^1
- f. the construction [$^{0}1$ $^{0}+$ [0 TheDenotatumOfIn $^{\tau}$ $^{0}g(D)$ $^{0}L^{1}$]] constructs nothing (because + obtains no suitable argument)
- g. the premise of the paradox, that D denotes certain N, is refuted
- strengthened or contingent versions make no counter-examples for the solution
- all known (and even unknown) principal paradoxes of denotation and reference are solved in (Raclavský 2011)

III.3 Explicit approach: solution to particular SPs (3.)

- quite analogously for various Liars which can be in fact rephrased to 'sentential'
 SPs of denotation and reference
- S: "S is not true" (rephrased: "S does not denote a true proposition")
- $-\lambda w\lambda t [^{0}\neg [^{0}\text{TrueIn}^{T}_{wt} ^{0}g(S) ^{0}L^{1}]]$ is not expressible in L^{1}
- consequently, S denotes a false proposition in L^2 (because there is no true^{π} proposition denoted by S in L^1)
- rejecting the premise of the Liar paradox that the proposition denoted by S can be true

III. Explicit approach: some important conclusions

- generally, all semantic concepts-constructions involving a construction of a korder code L^k are not expressible in L^k
- thus every code is limited in its expressive power

III. Explicit approach: some comparison

- it is a mix of 'golden' ideas of Russell (VCP, hierarchy of propositional functions),

 Tarski (language/metalanguage) and perhaps Kripke (partiality of truth-predicate)
- unlike Russell, TTT treats both 'extensional' and 'intensional' functions, the latter ones (constructions) being carefully individuated
- unlike Tarski, language is explicated as a system of expressions coding meaningsconstructions (which conform to the respective VCP) and semantic concepts are explicated as explicitly language relative
- unlike Kripke, semantic concepts in the total sense are explicated as well

IV. Implicit approach

- 1. Revenge
- 2. Disproving the revenge

IV.1 Implicit approach - an objection

- objection: as a solution to SPs this 'explicit approach' applies only to those paradox-producing expressions in which a language is explicitly mentioned (referred to); however, typical paradox-producing expressions need no disambiguation to the form in which a language is explicitly mentioned
- admitting the objection, I still claim that there is always at least *implicit* relativity to language (and that those terms are still ambivalent)
- in order to admit the objection, the following principle must be adopted

IV.1 Implicit approach - 'reducibility' principle

- something resembling to a Russellian Reducibility Principle (it follows from Tichý!s definition of his ramified type theory):

For every k+1-order construction of a property (relation) of expressions which involves a construction of a code L^k there is an equivalent lower-order construction constructing the same property (relation) but involving no construction of a code L^k .

- e.g., the 2nd-order construction

$$\lambda w \lambda t. \lambda n \left[{}^{0} \neg \left[{}^{0} \text{TrueIn}^{T}_{wt} n {}^{0} L^{1} \right] \right]$$

is equivalent to the 1st-order construction

$$\lambda w \lambda t. \lambda n \left[{}^{0} - \left[{}^{0} \text{True}^{\text{T}L1}_{wt} n \right] \right]$$
 (note "L1")

IV.1 Implicit approach - 'reducibility' principle (cont.)

- note that $\lambda w \lambda t. \lambda n \left[{}^{0} \left[{}^{0} \text{True}^{\text{T}L_{1}}{}_{wt} n \right] \right]$ is definable by means of $\lambda w \lambda t. \lambda n \left[{}^{0} \left[{}^{0} \text{TrueIn}^{\text{T}}{}_{wt} n {}^{0} L^{1} \right] \right]$
- there is a number of implicitly language relative semantic concepts definable this way

IV.1 Implicit approach: the danger of a revenge

- admitting the objection, "not true" without "in" expresses, in some code L^k , $\lambda w \lambda t . \lambda n$ [${}^0 \neg [{}^0 \text{True}^{\text{T}L1}_{wt} n]$]
- but there is a danger of a *revenge* (of a SP) if one accepts that "not true" expresses $\lambda w \lambda t. \lambda n \left[{}^{0} \left[{}^{0} \text{True}^{\text{T}L1}_{wt} n \right] \right]$ already in the 1st-order code L^{1}

- analogously for the other semantic terms and concepts, e.g. "Liar" (^oLiar is a 1st-order construction)

IV.2 Implicit approach: disproving the revenge

- Functional-Constructional VCP is incapable to preclude the revenge (though it works in the explicit case)
- I can appeal only to the *proof* quite analogous to those given by Tichý in Corollaries 44.1-4 that a k-order code cannot code (express by some its expression) construction like $\lambda w \lambda t. \lambda n \left[{}^{0} \left[{}^{0} \text{True}_{wt}^{\text{T}Lk} n \right] \right]$

(basically, S only seems but cannot express $\lambda w \lambda t [^0 \neg [^0 \text{True}^{\text{TL1}}_{wt} {}^0 g(S)]]$ in L^1 ; because: this construction does construct a total proposition P which is true if the proposition denoted by S in L^1 , say Q, is not true, and *vice versa*; i.e. P cannot be identical with Q, the alleged denotatum of S in L^1)

IV.2 Implicit approach – explanation (1.)

- as we have seen, constructions such as $\lambda w \lambda t \lambda n [^0 \neg [^0 \text{True}^{\text{TL1}}_{wt} n]]$ are definable by means of constructions explicitly utilizing the code L^1 ; thus the concept $\lambda w \lambda t . \lambda n [^0 \neg [^0 \text{True}^{\text{TL1}}_{wt} n]]$ is relative to language-code after all;
- indeed, $\lambda w \lambda t. \lambda n \left[{}^{0} \left[{}^{0} \text{True}^{\text{TL1}}_{wt} n \right] \right]$ and $\lambda w \lambda t. \lambda n \left[{}^{0} \left[{}^{0} \text{TrueIn}^{\text{T}}_{wt} n \, {}^{0} L^{1} \right] \right]$ construct one and the same property which is related to L^{1}

(all semantic properties and relations are relative to language)

IV.2 Implicit approach - explanation (2.)

- and on very natural assumptions, the purpose of any code is to discuss matters external to it
- it is not purpose of a code to discuss its own semantic features (Tichý 1988)
- once more, every code is limited in its expressive, coding power (Tichý 1988)

V. Concluding remarks

V. Concluding remarks

- the present approach to paradoxes resembles to hierarchical approaches of Russell and Tarski
- since it is a *neo-hierarchical* approach, its shares *pros* but avoids *cons* of Russell and Tarski; but the full defence of the present approach was not undertaken here
- but some important part (the Tichý's one) is not hierarchical, though it is Tarskian: the paradoxes are used in a proof that semantic concepts relating to a language are inexpressible in the very same language
- on the other hand, the diagnosis is very Russellian and Tarskian in spirit:

"As two contradictory statements are never both true, there cannot be any genuine paradoxes. Every apparent paradox is only a symptom of a hidden error." (Tichý 1988: 232)

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