Explication of Truth
in Transparent Intensional Logic

Logika: systémový rámec rozvoje oboru v ČR a koncepce logických propedeutik pro mezioborová studia (reg. č. CZ.1.07/2.2.00/28.0216, OPVK)

doc. PhDr. Jiří Raclavský, Ph.D. (raclavsky@phil.muni.cz)
Department of Philosophy, Masaryk University, Brno
Abstract

The approach of Transparent Intensional Logic to truth, which I develop here, differs significantly from rivalling approaches. The notion of truth is explicated by a three-level system of notions whereas the upper-level notions depend on the lower-level ones. Truth of possible world propositions lies in the bottom. Truth of hyperintensional entities – called constructions – which determine propositions is dependent on it. Truth of expressions depends on truth of their meanings; the meanings are explicated as constructions. The approach thus adopts a particular hyperintensional theory of meanings; truth of extralinguistic items is taken as primary. Truth of expressions is also dependent, either explicitly or implicitly, on language (its notion is thus also explicated within the approach). On each level, strong and weak variants of the notions are distinguished because the approach employs the Principle of Bivalence which adopts partiality. Since the formation of functions and constructions is non-circular, the system is framed within a ramified type theory having foundations in simple theory of types. The explication is immune to all forms of the Liar paradox. The definitions of notions of truth provided here are derivation rules of Pavel Tichý’s system of deduction.
I.1 Introduction: truth and logic

- Tarski’s seminal results in (1933/1976)
- dropping the “old fashioned” principle of bivalence by Kripke (1975) and others (partiality/trivalence)
- various rather non-classical approaches, e.g. Priest 1987 (dialetheism, paraconsistency), Gupta & Belnap 2004 (revisionism, four-values), Field 2008 and Beall 2009 (paracompleteness)
- recently, axiomatic approaches (Halbach 2011, Horsten 2011) are contrasted with the (older) semantic ones
- in the present paper, certain “neo-classical” approach is offered; truth is primarily property of extra-language items (“propositions”; => correspondence with facts); truth of expressions is derivative, depending also on language
I.2 Introduction: Transparent Intensional Logic

- the logical framework developed by Pavel Tichý from early 1970s
- semantic doctrine, i.e. logical explication of natural language meanings with many successful applications (see esp. Tichý 2004 – collected papers, Tichý 1988, recently Duží & Jespersen & Materna 2010)
- within TIL, semantic concepts are explicated as inescapably relative to language (Raclavský 2009, 2012), thus also the concept of language is explicated (ibid.); paradoxes are solved (a recourse to the fundamental truism that an expression $E$ can mean / denote / refer to something only relative to a particular language)
- as regards truth, three definitions by Tichý (1976, 1986, 1988) were elaborated in (Raclavský 2008, 2009, 2012)
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VII. Truth of expressions implicitly relative to language;
     solution to a revenge problem and conclusion.
II. TIL basics

- objects, functions and constructions
- deduction
- type theory
II. TIL basics: functions and constructions

- two notions of function (historically):
  a. as a mere mapping (‘graph’), i.e. function in ‘extensional sense’,
  b. as a structured recipe, procedure, i.e. function in ‘intensional sense’

- Tichý treats functions in both sense:
  a. under the name *functions*,
  b. under the name *constructions*

- an extensive defence of the notion of construction in Tichý 1988
II. TIL basics: objects and their constructions

- constructions are structured abstract, extra-linguistic procedures

- any object $O$ is constructible by infinitely many equivalent
  (more precisely $v$-congruent, where $v$ is valuation),
  yet not identical, constructions (=‘intensional’ criteria of individuation)

- each construction $C$ is specified by two features:
  i. which object $O$ (if any) is constructed by $C$
  ii. how $C$ constructs $O$ (by means of which subconstructions)

- note that constructions are closely connected with objects
II. TIL basics: kinds of constructions

- five (basic) kinds of constructions (where \( X \) is any object or construction and \( C_i \) is any construction; for exact specification of constructions see Tichý 1988):

  a. variables \( x \) ('variables')
  
b. trivializations \( ^0X \) ('constants')
  
c. compositions \( [C C_1\ldots C_n] \) ('applications')
  
d. closures \( \lambda xC \) ('\( \lambda \)-abstractions')
  
e. double executions \( ^2C \) (it \( v \)-constructs what is \( v \)-constructed by \( C \))

- definitions of subconstructions, free/bound variables ...

- constructions \( v \)-constructing nothing (c. or e.) are \( v \)-improper

- recall that constructions are not formal expressions; \( \lambda \)-terms are used only to denote constructions which are primary
II. TIL basics: deduction and definitions

- Tichý’s papers on deduction (though only within STT) in 2004
- because of partiality, classical derivation rules are a bit modified (but not given up)

- match $X:C$ where $X$ is a (trivialization of $O$), variable for $Os$ or nothing and $C$ is a (typically compound) construction of $O$

- sequents are made from matches

- derivation rules are made from sequents

- note that derivation rules exhibit properties of (and relations between) objects and their constructions (Raclavský & Kuchyňka 2011)

- viewing definitions as certain $\iff$-rules (ibid.); explications
II. TIL basics: simply type theory (STT)

- (already in Tichý 1976; generalized from Church 1940): let $B$ (basis) be a set of pairwise disjoint collections of objects:
  a. every member of $B$ is an (atomic) type over $B$
  b. if $\xi, \xi_1, ..., \xi_n$ are types over $B$, then $(\xi\xi_1...\xi_n)$, i.e. collection of total and partial functions from $\xi_1,...,\xi_n$ to $\xi$, is a (molecular) type over $B$

- for the analysis of natural discourse let $B_{TIL} = \{i,o,\omega,\tau\}$, where $i$ are individuals, $o$ are truth-values ($T$ and $F$), $\omega$ are possible worlds (as modal indices), $\tau$ are real numbers (as temporal indices)
- functions from $\omega$ and $\tau$ are intensions ($propositions$, $properties$, $relations-in-intension$, $individual~offices$, ...)

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II. TIL basics: semantic scheme

- somewhat Tichýan semantic scheme (middle 1970s):

  an expression $E$
  
  $| \quad \text{expresses (means) in } L$

  a construction $= \text{the meaning of } E \text{ in } L$

  $| \quad \text{constructs}$

  an intension / non-intension

  $= \text{the denotatum of } E \text{ in } L$

- the value of an intension in possible world $W$ at moment of time $T$ is the referent of an empirical expression $E$ ("the Pope", "tiger", "It rains") in $L$; the denotatum and referent (in $W$ at $T$) of a non-empirical expression $E$ ("not", "if, then", "3") are identical
II. TIL basics: example of logical analysis

“The Pope is popular” - the expression $E$

- $E$ expresses:

$$\lambda w \lambda t \left[ \text{Popular}_{wt} \circ \text{Pope}_{wt} \right]$$ - the construction of $P$

the procedure consists in taking a. the property “popular”, b. applying it to $W$ and $T$, c. getting thus the extension of “popular”, and then d. taking Pope”, e. applying it to $W$ and $T$, f. getting thus the individual who features that role, and g. asking whether he is in that extension – yielding thus $T$ or $F$ or nothing; analogously for the other $Ws$ and $Ts$ (abstraction)

- $E$ denotes, $C$ constructs:

...  

- $E$ refers in $W_1$ and $T_1$ to:

$$T$$ - the truth-value $T$
II. TIL basics: hyperintensionality

- intensional and sentencialistic analyses of belief sentences are wrong
- Tichý: belief attitudes are *attitudes towards constructions* of propositions (not to mere propositions or expressions); i.e. the agent believes just the construction expressed by the embedded sentence
- e.g., “Xenia believes that the Pope is popular” expresses the 2nd-order construction:

\[ \lambda w \lambda t [^0 \text{Believe}_{wt} ^0 \text{Xenia} \quad ^0 \lambda w \lambda t [^0 \text{Popular}_{wt} ^0 \text{Pope}_{wt}]] \]

(note the role of the trivialization; \((\alpha_1^*)_{\tau w}\)); another example: “X calculates 3÷0” expresses \( \lambda w \lambda t [^0 \text{Calculate}_{wt} ^0 X \quad ^0 [^0 3 ^0 ÷ 0]] \)
II. TIL basics: ramification of type theory (TTT)

- treatment of constructions inside the framework necessitates the ramification of the type theory

- cf. precise definition of TTT in Tichý 1988, chap. 5

  1. STT given above = first-order objects

  2. first-(second-, ..., n-)order constructions (members of types *₁, *₂, ..., *ₙ)

     = constructions of first-(second-, ..., n) order objects (or 2nd-, ..., n-1-order constructions)

  3. functions from or to constructions

- (Church like) cumulativity: every k-order construction is also a k+1-order construction

- ‘speaking about types’ in TTT with a basis richer than B_{TIL}
II. TIL basics: four Vicious Circle Principles (VCPs)

- understanding TTT as implementing four *Vicious Circle Principles* (VCPs), e.g. Raclavský 2009

- each of them is a result of the *Principle of Specification*: one cannot specify an item by means of the item itself

- *Functional VCP*: no function can contain itself among its own arguments or values (cf. STT, 1.)

- *Constructional VCP*: no construction can (v-)construct itself e.g., a variable c for constructions cannot be in its own range, it cannot v-construct itself (cf. RTT, 2.)

- *Functional-Constructional VCP*: no function F can contain a construction of F among its own arguments or values (cf.3)

- *Constructional-Functional VCP*: no construction C can construct a function having C among its own arguments or values (cf. 2. and 3.)
II. TIL basics: some conclusions about the approach

- unlike rivalling approaches:
  
  • it is explicitly stated what meanings are (meanings are constructions;
  
  • this semantical theory is hyperintensional (not intensional or extensional), i.e. its underlying TT is ramified;
  
  • the semantics and deduction go hand in hand;
  
  • the system is rather classical: bivalency and classical logical laws are accepted, yet it treats partiality (thus logical laws are a bit corrected);
  
  • the approach is *rather general*, it treats many logical phenomena (it is not a logic designed to a single, particular problem); the aim to explicate our whole conceptual scheme;
  
  • the overall feature is its objectual (not formalistic) spirit
III. Truth – three kinds of concepts

- truth of propositions / constructions / expressions
- language dependent / independent truth
III. Truth: three kinds of concepts

- the two truth-values \( T \) and \( F \) are not definable because every attempt of defining presupposes them; \( T \) and \( F \) represent affirmative and negative quality (Tichý 1988, 195)

- the concepts of truth due to their applicability to

  a. propositions  (2 kinds)
  b. constructions  (4 + 1 kinds)
  c. expressions  (6 principal kinds)

- *language independent*: a. and b.
- *language dependent*: c.

- b. is defined in terms of a.; c. is defined in terms of b. (and language, of course)
III. Truth of propositions (1.)

- the *bivalence principle* here adopted: for any proposition $P$, $P$ has at most one of the 2 truth-values $T$ and $F$ (in $W$ at $T$)

- i.e. proposition can be gappy (having no value differs from having a third-value);
  - e.g. “The king of France is bald” is gappy in the actual world and present time

- propositions are “built from” worlds, times and truth-values

- for a proposition to be true in $W$ at $T$ is nothing but simply having the truth-value $T$ as a functional value for the given argument (a $<W,T>$-couple), i.e. there is no mystery

- (philosophical issue: worlds of facts; facts explicated as propositions; indirect correspondence between propositions and facts)
III. Truth of propositions (2.)

- \((p\) ranges over propositions, \(o\) over the truth-values \(T\) and \(F\))

\[
[0^{\text{True}}_{\text{wt}^\pi} p] \iff^o p_{\text{wt}}
\]

= \textit{partial concept} of truth: a proposition \(P\) can be neither \text{true}^\pi \text{ or false}^\pi \text{ (the definiendum/definiens does not yield } \text{T or F for actually gappy propositions; deflationism; object-language)}

\[
[0^{\text{True}}_{\text{wt}^\pi} p] \iff^o [0 \exists \lambda o \left[ [o^0 = p_{\text{wt}}] \wedge [o^0 = \text{T}] \right]]
\]

= \textit{total concept} of truth: a proposition \(P\) is definitely \text{true}^\pi \text{ or not; (Tichý 1982, 233 – a congruent definiens); it is suitable for classical laws – e.g. “every proposition is true}^\pi \text{ or false}^\pi” \text{ (Raclavský 2010)}
III. Truth of propositions (3.)

- possible definitions of (some) non-classical connectives of three-valued logic
  (Raclavský 2010); for instance, whereas $\neg$ is a weak (partial) negation, the strong (total) negation (i.e. denial) is definable as

$$[{^0}\text{Denial}_{\text{wt}}^\pi p] \leftrightarrow^o [^0\neg[{^0}\text{True}_{\text{wt}}^\pi T p]]$$
IV. Truth of constructions
IV. Truth of constructions (1.)

- \( (c^k \text{ ranges over } k\text{-order constructions; the function } \Gamma^{v(\xi_k)} \text{ maps } C^k \text{ to the } \xi\text{-object, if any}, (v\text{-})constructed by } C^k \text{ – since there is a dependence on valuation, } 2 \text{ is a better choice, though it increases the order):} \)

\[
[F^{0}_{\text{True}}^{k_0}]_{0} \leftrightarrow [\exists \lambda o \left( [o^0 = 2^{c^k}]^0 \land [o^0 = 0T] \right)]
\]

(constructions \(v\)-constructing \(T\) may be called \(\textit{truths}\), constructions \(v\)-constructing \(T\) for any valuation \(v\) may be called \(\textit{L-truths}\); analogously for expressions, below)

- \( \textit{truth of constructions possibly constructing propositions} \) (4 kinds; on the next slide):
IV. Truth of constructions (2.)

- \([^{0}{\text{True}}_{\text{wt}}^{*\text{kP}} c^{k}] \iff^{o} [^{0}{\text{True}}_{\text{wt}}^\pi 2 c^{k}]\)  \((\text{partial})\)

- \([^{0}{\text{True}}_{\text{wt}}^{*\text{kPT}} c^{k}] \iff^{o} [^{0}{\text{True}}_{\text{wt}}^\tau 2 c^{k}]\)  \((\text{partial-total})\)

(constructions of, say, numbers, do not receive a truth value because the property ‘(BE) TRUE\(^T\) PROPOSITION’ does not apply to numbers)

- \([^{0}{\text{True}}_{\text{wt}}^{*\text{kT}} c^{k}] \iff^{o} [^{0}\exists \lambda o [ [o^{0}= 2 c^{k}]^{0} \land [o^{0}= 0 T] ]]\)  \((\text{total})\)
V. Truth of expressions
- incl. hierarchy of codes
V. Truth of expressions

- truth of expressions is relative to (dependent on) language(s)

- in what follows, this truism is incorporated in definitions of the following form

  (which seems also intuitively correct):

  an expression $E$ is true in $L$ ($W$ at $T$) iff it is true (in $W$ at $T$) what the expression $E$ means (= construction) in $L$

- compare: due to Tarski, an expression $E_1$ is true – tacitly presupposing in language $L_2$ -, if its translation $E_2$, i.e. the translation of $E_1$ from $L_1$ to $L_2$, is true, in $L_2$; Tarski thus presupposes the notion of translation; on natural construal, translatability means sameness of meaning; unlike Tarski, the approach advocated here explicate the notion of meaning)
V. Truth of expressions: hierarchies of codes (1.)

- what is language? language is a (normative) system enabling speakers to communicate (exchange messages)
- restricting here rather to the model of its coding means, i.e. language is *explicated* as function from expressions to meanings

- a *k*-order code $L^k$ is a function from real numbers (incl. Gödelian numbers of expressions) to *k*-order constructions, it is an $(*_k \tau)$-object (Tichý 1988, 228)
- there are various 1st-, 2nd-, ..., *n*-order codes (Tichý 1988)
V. Truth of expressions: hierarchies of codes (2.)

- it is not sufficient, however, to model coding means of (say) English by a single (say 1st-order) code
- rather, whole hierarchy of codes $L^1, L^2, ..., L^n$ should be utilized as a model of (say) English
- key reason: English is capable to code (express by some its expression) constructions of higher orders
V. Truth of expressions: hierarchies of codes (3.)

(realize that, e.g. [...c^1...] where c^1 is a variable for 1-order constructions is a 1+1-order construction, thus it cannot be coded by a 1-order code, only by 1+k-order ones)

- any construction of L^1, most notably ^0L^1, is among constructions not expressible in L^1 (recall Functional-Constructional VCP: if ^0L^1 would be a value of L^1, the function, L^1 were not be specifiable)
- ^0L^1 is the meaning of the name of L^1, i.e. “L^1”
- remember that every code is limited in its expressive power:
  a. no construction of a k-order code L^k is codable in L^k (only in a higher-order code)
  b. no expression mentioning (precisely: referring to) L^k is endowed with meaning in L^k (only in a higher-order code)
V. Truth of expressions: hierarchies of codes (4.)

- a hierarchy of codes involves (= conditions on hierarchies):

1. \( n \) codes of \( n \) mutually distinct orders

2. each expression \( E \) having a meaning \( M \) in \( L^k \) has the same meaning \( M \) in \( L^{k+m} \)

3. an expression \( E \) lacking meaning in \( L^k \) can be meaningful in \( L^{k+m} \)

- of course, most of the everyday communication takes place in the 1st-order code \( L^1 \) of the hierarchy; higher-order coding means (e.g., \( L^2 \)) are invoked rarely
V. Truth of expressions: hierarchies of codes (5.)

- few technical remarks:

- every code of the same hierarchy shares the same expressions, quantification over all of them is unrestricted

- due to order-cumulativity of functions, every k-order code is also a k+1-order code; thus the type \((*^n\tau)\) includes (practically) all codes of the hierarchy; we can quantify over them

- a hierarchy of codes is a certain class (it is an \((o(*^n\tau))\)-object); one can quantify over families

- note that a hierarchy of codes is a ‘system’ of coding vehicles, not a particular vehicle (‘language’); thus we investigate meanings of expressions in members of the hierarchy, e.g. \(L^n\), not in the hierarchy as a whole
V. Truth of expressions explicitly relative to language (1.)

- (language relative, 6 principal kinds; \([l^n n]\) v-constructs the value of an \(n\)-order code \(L^n\) for the expression \(E\), i.e. \(E\)'s meaning in \(L^n\)):

\[
[0^{\text{TrueIn}^P_{\text{wt}}} n l^n] \iff^o [0^{\text{True}^*_{\text{wt}}} [l^n n]] \tag{partial}
\]

\[
[0^{\text{TrueIn}^\text{PT}_{\text{wt}}} n l^n] \iff^o [0^{\text{True}^*_{\text{wt}}} [l^n n]] \tag{partial-total}
\]

(expressions denoting, in a given \(L\), propositions receives \(T\) or \(F\), all other expression receives nothing at all receives \(F\))

\[
[0^{\text{TrueIn}^T_{\text{wt}}} n l^n] \iff^o [0\exists\lambda o \left[ o^0 = 2[l^n n]_{\text{wt}} \right] o^0 = 0^T ] ] \tag{total}
\]

(thus all expressions denoting, in a given \(L\), non-propositions or nothing at all receive \(F\) as well as expressions denoting false or gappy propositions; an analogue is in Tichý 1988, 229)
V. Truth of expressions explicitly ... and immunity to the Liar paradox

- S: “S is not true”

  a) one should thus *disambiguate* S to (say) “S is not true in $L^1$”
  b) we understand S by means of (say) the 2nd-order code $L^2$ (or $L^3$, ...) of English
  c) such S means in $L^2$ the 2nd-order construction $\lambda w \lambda t [^0 \neg [^0 \text{TrueIn}_{^w t} ^0 g(S) ^0 L^1]]$
  d) being a 2nd-order construction, it cannot be expressed by S already in the 1st-order code $L^1$; thus S is meaningless in $L^1$
  e) lacking meaning in $L^1$, S cannot be true in $L^1$
  f) the premise of the paradox, that S can be true, is refuted
V. Truth of expressions explicitly ... and immunity to the Liar paradox (2.)

- non-strengthened or contingent versions to the Liar paradoxes make no counter-examples for the solution (Tichý 1988, Raclavský 2009a)
- analogously, the approach is immune to any semantic paradox (Raclavský 2012)
- for instance, the Paradox of Adder (D: ‘1 + the denotatum of D’) can be solved by the entirely analogous sequence of steps a)-f) as the Liar paradox
- explication to other semantic notions is analogous (and language relative):

\[
\begin{align*}
[0^\text{The Meaning Of In}^n \ n \ l^n] & \iff_{*n} [l^n \ n] \\
[0^\text{The Denotatum Of In}^\xi \ n \ l^n] & \iff_{\xi} [2[l^n \ n]] \\
[0^\text{The Referent Of In}^\kappa_{\text{wt}} \ n \ l^n] & \iff_{\kappa} [2[l^n \ n]_{\text{wt}}]
\end{align*}
\]
VI. Truth of expressions implicitly relative to language
VI. Truth of expressions implicitly relative to language
- above, only the concept ...E ... TRUE IN... L... was explicated
- each of such concepts is *explicitly* relative to language
- it was explicated as determining a *relation* (in intension)

- except that, there is a range of concepts ...E ... TRUE
- each of them determines a *property*
- these concepts are *implicitly* relative to language(s); they have to be disambiguated to the form ...E ... TRUE_Li
VI. Truth of expressions implicitly ... and explanation

- how to explain the impossibility?

- recall that constructions such as $\lambda w \lambda t. \lambda n \left[ 0 \neg \left[ 0 \text{True}_{L^1}^{T} w t n \right] \right]$ are definable by means of constructions explicitly utilizing the code $L^1$; thus the concept $\lambda w \lambda t. \lambda n \left[ 0 \neg \left[ 0 \text{True}_{L^1}^{T} w t n \right] \right]$ is relative to language-code after all;

the two constructions construct one and the same property which is related to $L^1$ (in a word, semantic properties and relations are relative to language)

- on very natural assumptions, the purpose of any code is to discuss matters external to it

- it is not purpose of a code to discuss its own semantic features (Tichý 1988, 232-233)

- once more, every code is limited in its expressive, coding power (ibid.)
VI. Truth of expressions implicitly ... and definitions
- the concepts of truth of expressions which are implicitly relative to language can be defined by $k$+1-order definitions of the form:

$$\left[{}^{0}\text{True}^{L_k}_{wt} n\right] \iff \circ \left[{}^{0}\text{TrueIn}_{wt} n {}^{0}L^{k}\right]$$

- here, $^0L^{k}$ cannot be weakened to $l^{k}$
- note the difference between $^0\text{True}^{L_k}$ and $^0\text{True}^{L^k}$ (relativity to $L^k$, not $L^{k}$)
- the construction $^0\text{True}^{L_k}$ (i.e. $\eta$-reduced form of $\lambda w\lambda t.\lambda n \left[{}^{0}\text{True}^{L_k}_{wt} n\right]$) is already $k$-order construction
- both $^0\text{True}^{L_k}$ and $\lambda w\lambda t.\lambda n \left[{}^{0}\text{TrueIn}_{wt} n {}^{0}L^{k}\right]$ construct one and the same property of expressions which is related to $L^{k}$
VI. Truth of expressions implicitly ... and a revenge?

- “not true” (without “in”) may express, in some code $L^k$, $\lambda w \lambda t. \lambda n [^0 \neg [^0 \text{True}^{TL_k}_{wt} n]]$, but then $k$ should be $>1$

- for there is a danger of a revenge of a Liar paradox if one accepts that “not true” expresses the construction $\lambda w \lambda t. \lambda n [^0 \neg [^0 \text{True}^{TL_k}_{wt} n]]$ already in the $k$-order code $L^k$

- (it is a fact that for every $k+1$-order construction of a property/relation of expressions which involves a construction of a code $L^k$ there is an equivalent lower-order construction constructing the same property/relation but involving no construction of a code $L^k$)
VI. Truth of expressions ...: disproving the revenge

- Functional-Constructional VCP is incapable to preclude the revenge (as it does in the explicit case)

- I can appeal only to the proof - given by Tichý 1988 as Corollaries 44.1-3 - that a $k$-order code cannot code (express by some its expression) constructions like $\lambda w \lambda t. \lambda n [^0 \rightarrow [^0 \text{True}_{^w t}^{^L k} n]]$

- the idea of the proof: the sentence $S$ only seems but cannot express $\lambda w \lambda t [^0 \rightarrow [^0 \text{True}_{^w t}^{^L 1} g(S)]]$ in $L^1$; because: this construction does construct a total proposition $P$ which is true if the proposition denoted by $S$ in $L^1$, say $Q$, is not true, and vice versa; i.e. $P$ cannot be identical with $Q$, the alleged denotatum of $S$ in $L^1$
VII. Concluding: Tarski’s Undefinability Theorem

- Tarski’s Undefinability Theorem (UT) says that semantic predicates concerning $L$ are not definable in $L$
- the TIL-approach fully confirms UT
- only one correction: the concepts are definable (the constructions exist and they even construct something), but they cannot be expressed in $L$
VII. Concluding: languages with limited expressive powers

- note that a partial truth-predicate can be added only to that object language-code which is of a *limited expressive power* (natural language is not such), i.e.

1) language not allowing to form a total untruth-predicate from the partial truth-predicate, or

2) language not containing any equivalent of the total untruth-predicate, e.g.:

\[
[0_{\text{Babig}_{wt}} n] \leftrightarrow^o [0_{\neg[0_{\exists \lambda o \left[ o^o = \left[0_{\text{TrueIn}_{wt}^p n L^1} \right]^0 \land \left[ o^o = 0_T \right] \right]}]}]
\]
VII. Some final conclusions about the proposal (1.)

- sharp contrast between truth-predicate and truth-concept (i.e. the construction expressed by the predicate), which leads to the supplementation of UT

- underlining the contrast between truth of expressions (which is language-dependent) and truth of semantic contents (which is language-independent), which philosophically welcome

- truth of propositions is clear and uncontroversial

- truth of expressions clearly depends on their meanings

- the difference between truth of expressions explicitly / implicitly relative to language
VII. Some final conclusions about the proposal (2.)

- both *total and partial variants* (with implications for issues in philosophy of logic) of truth predicates/concepts

- implications for the correspondence theory (facts, ...), see Kuchyňka and Raclavský (2014)
Key references

References


