From Frege’s Semantic Triangle to the Semantic Square of Transparent Intensional Logic

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Logika: systémový rámec rozvoje oboru v ČR a koncepce logických propedeutik pro mezioborová studia (reg. č. CZ.1.07/2.2.00/28.0216, OPVK)

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Abstract

The talk introduces the semantic scheme of Pavel Tichý's hyperintensional semantics as a scheme of four notions: expression – meaning – denotation – reference. The motivation for such fine-grained semantics is exposed and the reason for adopting each particular notion is demonstrated on logical analyses of identity statements and their logical consequences. We begin with Frege’s refutation of ‘one-legged’ semantic scheme, which was replaced by his ‘two-legged’ semantic scheme, the famous Frege’s triangle. The attempt to interpret vertices of the triangle within intensional semantics and logic is rejected because of hyperintensional context. We show that Tichý’s semantic square is more faithful to semantic ideas of Frege (of course, provided reference of empirical and mathematical expression is distinguished and set apart, which means that one vertex of a triangle is split into two vertices). We show that Tichý’s semantical system can escape an important criticism of Frege’s notion of sense.

Structure of the talk

I. Approaching the semantic square: two semantic triangles

1. Frege's triangle
2. Russell's 'robust realism'
3. Intensions of possible world semantics

II. The semantic square of hyperintensional semantics

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I.

Approaching the semantic square: two semantic triangles

1. Frege's triangle

2. Russell's 'robust realism'

3. Intensions of possible world semantics

I.1. Frege's triangle

I.1. Identity statements and cognitive content

- let a schematic *identity statement* (IS) be

"x=y"

- examples:

"2+3=√25" - a mathematical/logical IS,

"The morning star = the evening star" - an empirical IS

- in his Begriffsschrift (1979), *G. Frege* met a challenging problem: what is the *cognitive content* of ISs?

I.1. Frege's puzzle: cognitive content and truth of identity statements

- Frege reopened the question in his landmark paper 'Über Sinn und Bedeutung' (1892) and observed that:

a) ISs are often *informative*, they bring a valuable, *nontrivial* piece of an *a posteriori* knowledge

b) but ISs must be somehow about *self-identity of an object*, otherwise they can't be *true at all* (if an IS is about distinct objects, it is simply contradictory)

c) self-identity follows from the Axiom of Identity (one of the core axiom of European metaphysics), the corresponding statement is thus analytic and the knowledge of an object's identity is *trivial*, *uninformative*

- Frege's puzzle consists in this *heterogeneous* set of *desiderata*

I.1. Frege’s puzzle: substitutivity (failure of the Leibniz Principle)

- Frege met not only *semantical*, but also *logical* problems concerning ISs
- the *Leibniz substitutivity principle (SI)* licences us to replace ‘identicals’ within formulas, i.e.:

(...*x*...), *x*=*y* |- (...[*y*/*x*]...) (some occurrences of *x* are replaced by *y*)

- Frege’s famous example (his choice: *x*=the morning star, *y*=the evening star) shows *failure* of SI:

“*A* believes that *x*=*x*”

“*x*=*y*”

“Therefore, *A* believes that *x*=*y*”

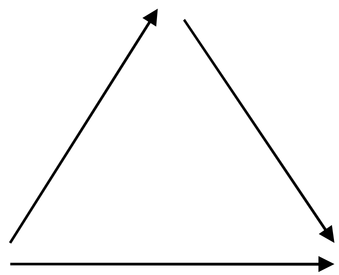
I.1. Frege's semantic scheme = Frege's triangle

- Frege thus had to substantially *revise* our *folk semantics* with its one-legged *semantic scheme* ($\longrightarrow$ = means/expresses/signifies/names):

name $\longrightarrow$ object

in favour of his two-legged *semantic scheme*:

Sinn (sense, today: meaning)



expression *Bedeutung* (meaning, today: denotation/reference)

- (important question: what is *Sinn* exactly? answer: an objective complex entity, perhaps compositional, which determines an object – a function?)

I.1. Frege's triangle - an application to Frege's puzzle

- the core of Frege's theory: the meaning of *E* is *split* and the *Sinn* of an expression *E* ≠ the *Bedeutung* of an expression *E*
- by splitting meaning of expressions, Frege was capable to *explain failure of SI*:
- i. in *direct* ("gerade") *contexts*, expressions are *about* (stand for) their *Bedeutung*s (denotata), SI is applicable
- ii. in *indirect* ("ungerade") *contexts* ("believes that...", "says that..."), expressions are *about* (stand for) their *Sinn*s, which is the reason why we cannot apply SI

- isn't there circularity in defining indirect context as contexts in which SI fails, while explaining failure of SI in terms of indirect contexts? (Tichý 1986)

I.1. Frege's triangle – further criticism

- (according to Frege) in direct contexts, an expression *E* stands for its *direct Bedeutung* and expresses it *direct Sinn*; in indirect contexts, an expression *E* stands for its *indirect Bedeutung* (= *direct Sinn*) and expresses its *indirect Sinn*
- thus one must map *E*'s *Bedeutung* to *E*'s 1st-level *Sinn*, *E*'s 1st-level *Sinn* to *E*'s 2nd-level *Sinn* etc., thus, there is *an infinite hierarchy of Sinn*s of one expression *E* in correspondence to the hierarchy of contexts
- how can an average speaker manage such infinite number of *Sinn*s? (Schiffer 1984)
- there is no 'backward road' mapping from *Bedeutung* to *Sinn* (Russell 1905)
- isn't there a *simpler* theory which can solve Frege's puzzle without evoking *Sinn*s? (yes: Russell's)

I.2. Russell's 'robust sense for reality'

I.2. Russell's extensionalistic reaction

- *Bertrand Russell* (1903 – *The Principle of Mathematics, Appendix A.*, 1905 – *On Denoting*, correspondence with Frege – the case of snow of Mt. Blanc):

a) not accepting *Sinn*s, R. recognized only *idea*, which is psychological; there are only names and objects (and classes/relations, propositions and p.f.s)

b) R. split the category of names to: i. *proper names*, ii. *descriptions ('the φ')*

c) R. declared that descriptions are *not* (!) *about* the referred objects:

"The F is G" reduces to $\exists x.\ F(x)\ \&\ G(x)\ \&\ \forall y.\ F(y)\equiv(y=x)$

d) the theory of descriptions treats well the identity contexts ("Walter Scott = the author of Waverley", "George IV wondered to know whether ...")

I.2. Church's fatal objection to Russell's theory

- A. Church, the follower of Frege's semantics (1951, 1951a), raised a fatal objection to Russell's theory of descriptions (not only that there are problems with negations of existential statements); his example:

"Ponce de Leon seeks the Fountain of Youth"

$$\exists x \text{ . IsTheFoY}(x) \text{ \& Seek(PdL, } x) \text{ \& } \forall y. \text{ IsTheFoY}(y) \equiv (y = x)$$

- Russell's elimination method allows us to derive an existential claim $\exists x$ [IsTheFoY(x)] ("There is the Fountain of Youth"), while the sentence has *no such existential import* = Russell's theory is thus provably *wrong*
- Kaplan (1975): Frege's theory is thus clearly *better* than Russell's

I.2. Extensionalistic reaction to both Frege and Russell

- Peter F. Strawson (1950 – *On Referring*) and *legions* of other theoreticians of *singular terms* (e.g. S.A. Kripke) reject Russell's crucial claim c)
- *direct reference* theorists (N.U. Salmon, etc.): a *Russellian proposition* involves an object (an individual) *directly* named by proper names / described by descriptions; there arises the problem of *New Frege's puzzle*

- *descriptivists* (Kripke 1972/1980 wrongly attributed descriptivism to Russell and Frege; metalinguistic theory, rigidified descriptions): to escape Frege's puzzle, the content of a proper name is a meaning of a description such that "$N =_{(df)}$ the F"
- but they exist also *neo-Fregeans* (e.g. G. Forbes, P. Tichý): attempts to propose modern models of *Sinn*

I.3. Intensions of possible world semantics

I.3. Possible world semantics and Frege's triangle (Carnap)

- Frege's pupil *R. Carnap* (1947/58) reinterpreted vertices of Frege's triangle:

$$\begin{tikzcd} & \textit{intension} \arrow[dr] & \\ \textit{expression} \arrow[ur] \arrow[rr] & & \textit{extension} \end{tikzcd}$$

- each expression has both extension and intension; but: the extension is preferable, the intension is chosen in the case of need (indirect contexts)
- thus, Carnap's Method of Intension and Extension brings the first explanation of Frege's *Sinn*: it is something *like* a function from state-descriptions to extensional objects

I.3. Possible world semantics and (post-)Frege's triangle

- in works of Kripke (1963) and others (Hintikka, Kanger, ...), Carnap's state-descriptions were turned into *possible worlds* (Ws)
- *possible world intensions* are defined as functions from possible worlds (or ⟨W,T⟩ couples):

propositions	(to truth-values)	
properties	(to classes of objects)	
n-ary *relations*	(to *n*-tuples of objects)	
individual concepts	(to individuals)	etc.

- Montague, Hintikka, (early) Tichý and others thus significantly *re*interpret Frege's triangle
- (discrimination between PWS-intension/extension helps to control validity of arguments)

I.3. Tichý's 1971 (and after) semantic doctrine

- Pavel Tichý (1936 Brno, Czechoslovakia – 1994 Dunedin, New Zealand)
- 'An Approach to Intensional Analysis' (1971, Noûs), the early version of his *Transparent intensional logic* (TIL), a modification of Church's (1940) *typed λ-calculus* (a 'higher order logic')
- according to Tichý (unlike Carnap or Montague), a descriptive term stands for one and the same *determiner* (= *intension*) in every - thus even in a transparent - context
- since determiners are explicated as intensions, they are represented by means of λ-terms binding a possible world variable:

$$\lambda w\ (\ldots w \ldots)$$

- $\lambda w\ (C\ w)$ η-contracts to C; let us write C_w instead of $(C\ w)$

I.3. Identity statements and (early Transparent) intensional logic

- let $\boldsymbol{D}$ and $\boldsymbol{D'}$ be the logical analyses of descriptions "D" and "D'" which denote determiners D and D', respectively; $\boldsymbol{D}$ is said to *yield* the *determiner* D
- $\boldsymbol{D}_w$ yields D's *determinee* in W (analogously for $\boldsymbol{D}_{w'}$, $\boldsymbol{D'}_w$, $\boldsymbol{D'}_{w'}$)
- important claim: *one schematic logical form x=y underlies multiple particular logical forms which differ in (so-called) hospitability of their variables*
- see the logic for x=y in 'Indiscernibility of identicals' (1986, Studia Logica):

... $\boldsymbol{D} = \boldsymbol{D'}$ ...	(identity between two determiners)
... $\boldsymbol{D}_w = \boldsymbol{D'}$ ...	(identity between a determinee in a W and a determiner)
... $\boldsymbol{D}_w = \boldsymbol{D'}_w$ ...	(identity between two determinees in a W)
... $\boldsymbol{D}_{w'} = \boldsymbol{D'}_w$ ...	(identity between a determinee in a W' and a determinee in W)

etc.

I.3. (Early Transparent) intensional logic and intensional contexts

- a) *modal contexts* ($\Box p =_{df} \forall\lambda w.p_w$; where p is a variable for propositions)

  ... $\Box\ (\lambda w.\ D_w = D'_w)$ ... (*de dicto*)

  ... $\Box\ (\lambda w.\ D_{w'} = D'_w)$ ... (*de re*)

- such careful treatments of scope immediately solves *tons* of modal puzzles

- b) *propositional attitudes/contexts* (A is an agent)

  ... $\lambda w'.\ \mathrm{Bel}_{w'} A\ (\lambda w.\ D_w = D'_w)$ ... (*de dicto*)

  ... $\lambda w'.\ \mathrm{Bel}_{w'} A\ (\lambda w.\ D_{w'} = D'_w)$ ... (*de re*)

- note that the logical analysis of "D is D'" (or any other expressions), i.e. $(\lambda w.\ D_w = D'_w)$, *does not change dependently on context*

- c) '*notional' attitudes/contexts* (Church's objection to Russell is avoided)

  ... $\lambda w'.\ \mathrm{Seek}_{w'} A\ D$ ...

I.4. Brief recapitulation of part I.

- Frege solved Frege's puzzle by splitting significance of expressions
- problems of Frege's theory include:
    i. suspicious infinite hierarchy of *Sinn*s and
    ii. unknown logical nature of *Sinn*
- PW-semantics (intensional logic) retains Frege's meaning dualisms by employing both intensions and extensions
- PW-semantics (intensional logic) provides a convenient logical model of *Sinn*, viz. PWS-intensions, which are such and such functions
- a suitable (e.g. Tichý's) intensional semantics/logic can treat nicely the hierarchy of intensions (cf. e.g. $((\mathrm{I}w)w)$ or $\lambda w.(\lambda w'(\mathrm{I}w'))\ w)$)
- discriminating between PWS-intension and extension *improved control over arguments* where empirical/non-empirical difference plays a role (cf. Quine)

II.

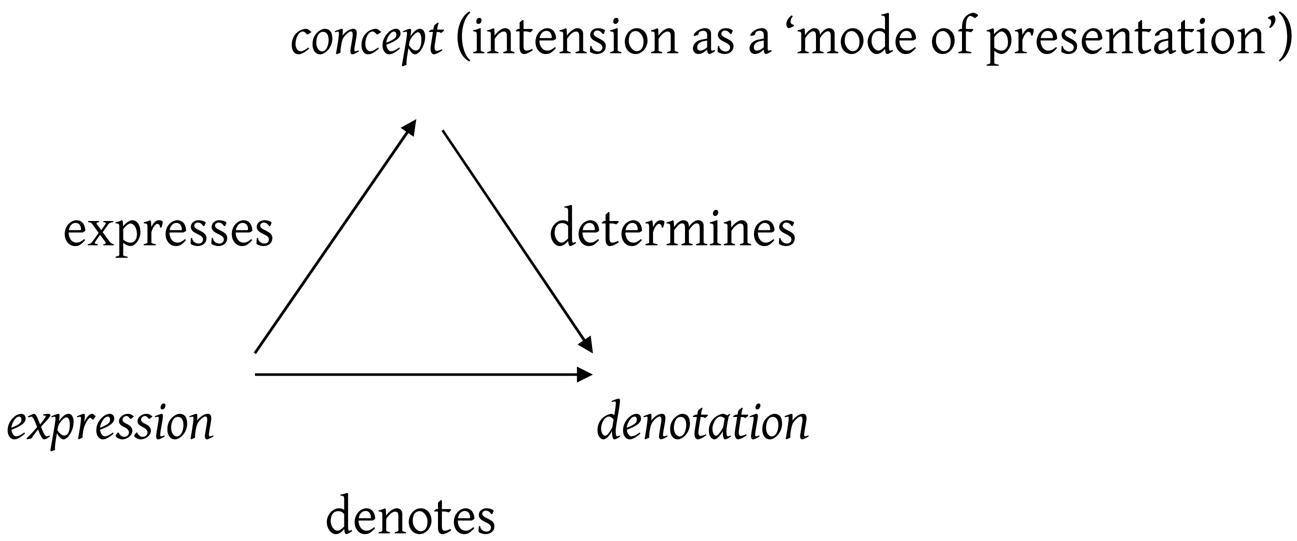
The semantic square of hyperintensional semantics

1. Weakness of possible world semantics / intensional logic
2. Hyperintensionality

II.1. Weakness of possible world semantics / intensional logic

II.1. Possible world semantics *vs.* Frege's triangle

- firstly note that the triangle of PW-semantics provides a *desintepretation* of Frege's triangle
- *logical modality* (modelled then by functional dependence on possible worlds) is entirely *unknown* or unmentioned by Frege and even Church
- Church's (1951) triangle treats rather *intensions* in the *older sense* of the word



II.1. Church's (and Thomason's) intensions vs. Frege's *Sinns*

- in Church's type theory, the *Sinn* for *X* (e.g. an individual of type ι_0) is a *logically primitive object* belonging to distinct *atomic* type (viz. ι_1)
- thus, the logical nature Church's *Sinn*s does not seem to match Frege's *Sinn*s
- the way *how* a *Sinn determines the corresponding object is entirely unknown* in the system (we can only ascribe to an ι_1-object that it does/does not determine an ι_0-object)
- Thomason's (1980) 'hyperintensional' semantics for propositional attitudes borrows this feature: it only provides links between PWS-propositions and objects of an atomic type (which can be written as) π and we do not know why and how a π-object relates to a particular proposition (nevertheless, this solution of the hyperintensionality problem is better than some rivalling ones)

II.1. The pursuit of hyperintensions (1/2)

- Lewis (1970) published a first serious criticism of possible world semantics (the *semantic argument*): the intuitive meanings has a more *fine-grained structure* than PWS-intensions
- Lewis (and others): let’s put intensions on (math.) trees, we get thus a structure
- Asher (1984): meanings don’t grow on the trees - a version of Church’s (1950) older objection to Carnap’s (1947/1958) notion of intensional isomorphism which makes meanings peculiar linguistic entities
- the *logical cohesion argument* (e.g. Tichý 1988): what binds an intension together with its argument - the tree? (a nonstarter: another function/relation – it leads to Bradley’s regress)

II.1. The pursuit of hyperintensions (2/2)

- the *'philosophical' argument*: modelling intentional attitudes as attitudes towards intensions is questionable on various grounds
- Cresswell (1975) talks about *hyperintensional contexts*, i.e. contexts, which cannot be tackled by adopting mere PWS-intensions; extended in (1985):
- the *logical inference argument*: PWS-semantics produces wrong inference results
- since *A* is understood as believing one and the same PWS-proposition, the following kind of intuitively invalid arguments is assessed as valid:

"*A* believes 2+3=2+3"

"2+3=√25"

"Therefore, *A* believes 2+3=√25"

II.2. Hyperintensionality

II.2. Hyperintensions

- we need an entity standing in semantic scheme *between* an expression and an intension/extension
- i.a) hyperintensions should be *distinct* from expressions (to avoid linguistic dependence of attitude ascriptions – in the sense of Church's criticism),
- i.b) but there should be a reasonable *correspondence* between expressions and hyperintensions
- ii.a) hyperintensions should be more *fine-grained* than an intensions / extensions,
- ii.b) while they should determine intensions/extensions (in order to be suitable objects for propositional attitudes etc.); the mechanism of *determining* has to be enough clear

II.2. Tichý's discovery of hyperintensions

- arguably, there are more hyperintensional semantics/logics (e.g. Cresswell 1985, Zalta 1988); I accept Tichý's *Transparent Intensional Logic* (*TIL*) whose supreme version occurs in (1988) (see also Duží et al. 2010 or Raclavský 2009)
- Tichý provided a *hyperintensional model of meaning already in 1978* (e.g. 'Two Kinds of Intensional Logic', Epistemologia)
- in early 1970s, Tichý realized that his λ-terms can be read as standing for
    - i. their denotation, i.e. 'extensional entities'
    - ii. their sense, i.e. 'intensional entities'
- this difference between *two kind of semantics* was recently emphasized by J.Y. Girard (with some accent on computing as a process)

II.2. Semantics of sense *vs.* semantics of denotation

"we have an *equality*

$$27 \times 37 = 999$$

This equality makes sense in the mainstream by saying that the two sides *denote* the same integer and that × is a *function* in the Cantorian sense of a graph.

This is the denotational aspect, which is undoubtedly correct, but it misses the essential point:

There is a finite *computation* process which shows that the denotations are equal. ... The two expressions ["27 × 37" and "999"] have different *senses* and we must *do* something (make a proof or calculation ...) to show that these two *senses* have the same *denotation*"

(Girard 2003, *Types and Proofs*, 1-2)

II.2. Tichý's constructions

- Tichý calls the 'intensional senses' of his λ-terms *constructions*
- in (1986) he says that he borrowed the term from geometry where one figure can be constructed various ways; they are akin to *algorithmic computations*
- *'intensional principle' of individuation*: every object (incl. a construction) is constructed by an infinite number *congruent*, but *not identical* constructions
- for example, the number 5 can be constructed by adding 3 to 2 or by calculating the square root of 25, *etc*. (including a trivial, immediate construction of 5)
- every construction *C* is thus *specified by*:

  i. the object *O* constructed by *C*

  ii. the way how *C* constructs the object *O* (by means of which subconstructions)

II.2. Modes of forming constructions

i. *Variable* x_k *v*-constructs the *k*-th object (of an appropriate type) of the valuation *v*.

ii. *Trivialization* ^0X *v*-constructs (for any *v*) the object (or construction) *X* directly, without any change.

iii. *Single execution* ^1X *v*-constructs the object (if any) *v*-constructed by *X*.

iv. *Double execution* ^2X v constructs the object (if any) which is *v*-constructed by the construction (if any) *v*-constructed by *X*;

v. *Composition* $[C\ C_1...C_n]$ *v*-constructs the value (if any) of the function *F* (if any) *v*-constructed by *C* on the string of entities A_1 ... A_n (if any) *v*-constructed by C_1, ..., C_n

vi. *Closure* λxC *v*-constructs (for any *v*) a function which maps the objects in the range of *x* to the objects which are *v*-constructed by *C* (a very much simplified formulation).

II.2. Tichý's ramified theory of types

Let *B* (base) be a non-empty class of pairwise disjoint collections of atomic objects, e.g. $B_{TIL}=\{\iota,o,\omega,\tau\}$.

(t.1) (***1st-order types***, i.e. types collecting 1st-order objects)

a) Any member of *B* is a *1st-order type over B*.

b) If α_1, ..., α_m, β are 1st-order types over *B*, then $(\beta\alpha_1...\alpha_m)$ – the collection of all total and partial *m*-ary functions from α_1, ..., α_m to β – is a *1st-order type over B*.

c) Nothing is a 1st-order type over B unless it so follows from (t.1).a)-b).

(tbc.)

II.2. Tichý's ramified theory of types (cont.)

(c.n) (***n*-order constructions**, i.e. constructions of n-order objects)

a)-b) Any variable x v-constructing an n-order object, the trivialization of any n-order object X, i.e. ${}^0X$, is an *n-order construction over B*.

c)-f) If x_1, ..., x_m, Y, X, X_1, ..., X_m are n-order constructions over B, then ${}^1X_{(i)}$, ${}^2X_{(i)}$, $[X X_1...X_m]$, and $\lambda x_1...x_m\ Y$ are *n-order constructions over B*.

g) Nothing is an n-order construction over B unless it so follows from (c.n).a)-f).

Now let $*_n$ be a type of n-order constructions.

(t.n+1) (***n+1-order types***)

a) $*_n$ and any n-order type over B is an n+1-*order type over B*.

b) If α_1, ..., α_m, β are n-order types over B, then $(\beta\alpha_1...\alpha_m)$ – the collection of all total and partial m-ary functions from α_1, ..., α_m to β – is an n+1-*order type over B*.

c) Nothing is an n+1-order type over B unless it so follows from (t.n+1).a)-b).

II.2. Constructions – a linguistic example

"Fido is a dog" equivalent expressions "Fido is not a non-dog"

| each expression expresses a construction |

$\lambda w \lambda t.\mathbf{Dog}_{wt}\ \mathbf{Fido}$ congruent constructions $\lambda w \lambda t.\neg.(\mathbf{Non\ Dog})_{wt}\ \mathbf{Fido}$

\ each construction constructs the proposition /

the PWS-proposition that Fido is a dog

|

True is the value of the proposition in some possible worlds (times)

II.2. Tichý's semantic square

- Tichý thus combines Frege's triangle and the triangle of PW-semantics

$$\begin{tikzcd}[column sep=huge, row sep=huge]
\text{meaning } \boldsymbol{M} \text{ (construction)} \arrow[r, "\textit{constructs}"] & \text{denotatum } \boldsymbol{D} \text{ (intension/extension)} \arrow[d, "\textit{determines}"] \\
\text{expression } \boldsymbol{E} \arrow[u, "\textit{expresses}"] \arrow[ur, "\textit{denotes}" near start] \arrow[r, "\textit{refers to in W (at T)}"'] & \text{referent } \boldsymbol{R} \text{ (the value of an intension in } W\text{)}
\end{tikzcd}$$

II.2. Constructions as *Sinns*

- constructions are apt models of *Sinn*s because they are:

  a. objective, abstract (and also independent of language, incl. λ-formalism)

  b. they can be suitable explicates of *meanings*, thus they are 'graspable by any speaker who masters the sign system in question'

  c. they are complex entities, they are 'compositional'

  d. they are 'modes of presentations' of objects (they construct objects)

  e. they form a hierarchy (a construction C is constructed by higher-order constructions)

- (Tichý 1986a, 1988 - 'backward road mapping' maps an object O to its trivial, thus intelligible, construction ^0O; let us write "$\mathbf{O}$" instead of "^0O")

II.2. Frege's puzzle and Tichý's 'constructional attitudes'

- Tichý (1988): 'propositional' attitudes cannot be attitudes towards PWS-propositions but towards *constructions of PWS-propositions*

"*A* believes 2+3=2+3"

"2+3=√25"

"Therefore, *A* believes 2+3=√25"

$\lambda w \lambda t.\ \mathbf{Bel}^k_{wt}\ A\ {}^0(\lambda w \lambda t.\ 2+3=2+3)$ - substitution in $(\ldots{}^0(\ldots)\ldots)$ is not allowed, though one can substitute in ${}^0(\ldots)$ (!!!)

$2+3=\sqrt{25}$ - *congruence* of constructions flanking =

$\lambda w \lambda t.\ \mathbf{Bel}^k_{wt}\ A\ {}^0(\lambda w \lambda t.\ 2+3=\sqrt{25})$ - an unrelated propositional construction

II.2. Constructions in identity statements

- in Tichý's *ramified* version of his type theory, constructions are explicitly treated, i.e. they can be quantified upon, substituted for one another etc.
- of course, constructions can be substituted also through the construction

$x=y$

- the substitutional 'behaviour' of constructions is *limited* by the type theory
- the rules controlling *hospitability* of constructional variables were not stated by Tichý and the issue is still an open problem
- anyway, Tichý's supreme version TIL extends our possibilities as regards $x=y$

III.

Conclusion

III. Conclusion

- Frege's puzzle led Frege to introduce the *semantic triangle*:

  '*expression-intension-extension*',

  which replaces the naïve, folk semantical theory
- possible world semantics (intensional logic) follows Carnap in understanding PWS-intension/extension as a tool for discrimination between *empirical and non-empirical* expressions (which positively affects a control of arguments)
- criticism of possible world semantics (intensional logic) leads to the refreshing of Frege's original intention of *Sinn* as an 'intensional' entity
- *hyperintensional semantics* (and logic) transforms the triangle into the *semantic square* by adopting *hyperintensions* (which positively affects control of arguments):

  '*expression-hyperintension-intension-extension*'

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