
A Light Introduction to Transparent Intensional Logic and its Application in Semantics



INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Logika: systémový rámec rozvoje oboru v ČR a koncepce logických propedeutik
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Abstract

The aim of the talk is to present the core of TIL, i.e. Pavel Tichý's Transparent intensional logic, and its essential applications. We explain its motivation (mainly hyperintensional phenomena) for adoption of the key notions of TIL and its crucial distinctions (e.g. constructions vs. functions). Then, we expose the core of the apparatus (constructions, type theory, deduction), explaining and commenting each its part. Finally, we move to applications, especially those in the field of natural language (analysis of meaning) and several important philosophical notions (e.g. conceptual systems).

Content

- I. Overview of TIL
- II. Introduction to main notions (the apparatus)
- III. Some applications of TIL

I. Overview

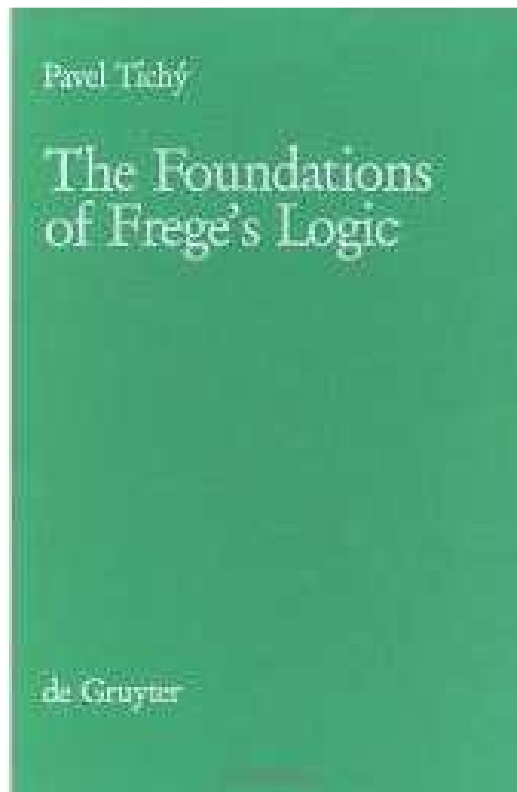
I. Overview: What is Transparent Intensional Logic (TIL)?

- *Pavel Tichý* (*1936 Brno, #1994 Dunedin), the originator of TIL
- rough characterizations of TIL:
 - a) - higher-order logic with intensions and procedures-algorithms
 - b) functions as mappings and functions as structured procedures are distinguished
 - c) procedures are (algorithmic) entities determining mappings
 - d) its λ -notation is not the primary target of study, but the procedures
 - e) the underlying type theory is ramified
- briefly, TIL is objectually understood λ -calculus within (ramified) theory of types

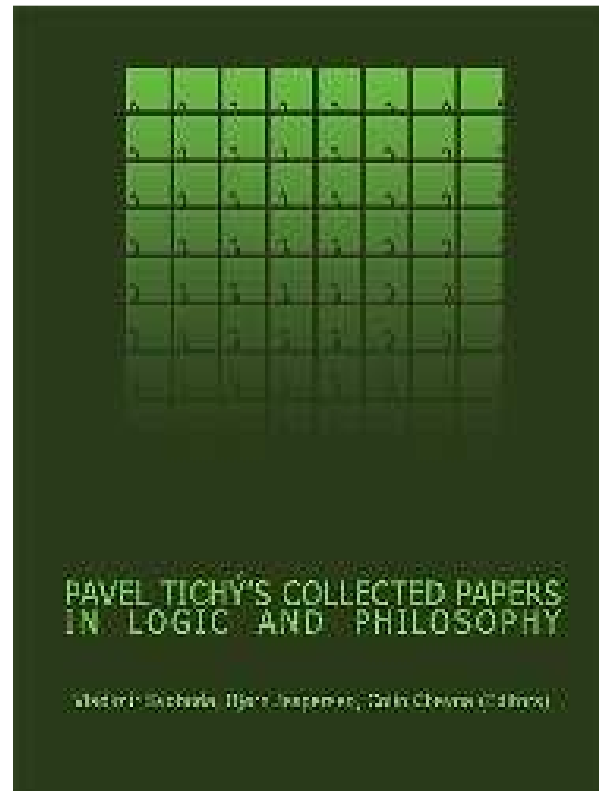
I. Overview: TIL vs. rest of Tichý's views

- TIL \neq Tichý's theory of types
- TIL is a particular instance of Tichý's theory of types
(namely, it that instance of Tichý's TT which has basis containing the type of individuals, truth-values, possible worlds and time-moments)
- TIL \neq semantical and logical doctrines of Tichý or TILians
- one can use TIL without utilizing those semantical and logical doctrines (e.g. on can maintain that meanings of sentences are not constructions of propositions)

I. Overview: Classical source of information on TIL



Tichý (1988) a book not only about Frege, the final version of the apparatus

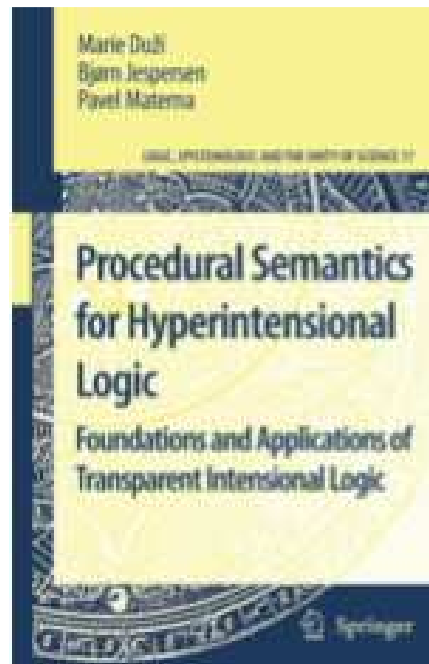


Tichý (2004) – collected papers: many applications of TIL

I. Overview: Where to get more information about TIL?

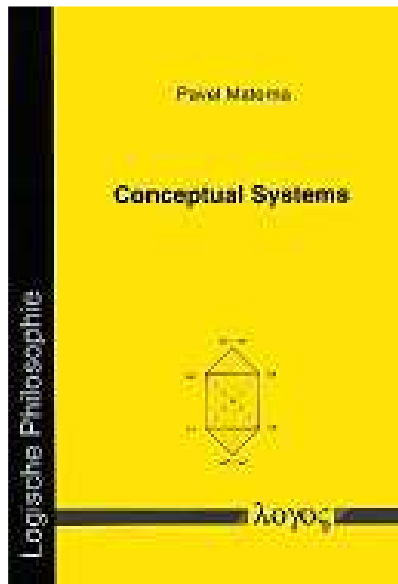


Raclavský (2009), in Czech; an extensive use of TIL to semantics of names and descriptive phrases (includes also, e.g., solution to semantic paradoxes)



(2010) a codification of **Pavel Materna's**, **Marie Duží's** and **Bjørn Jespersen's** version of TIL

I. Overview: Where to get more information about TIL? (cont.)

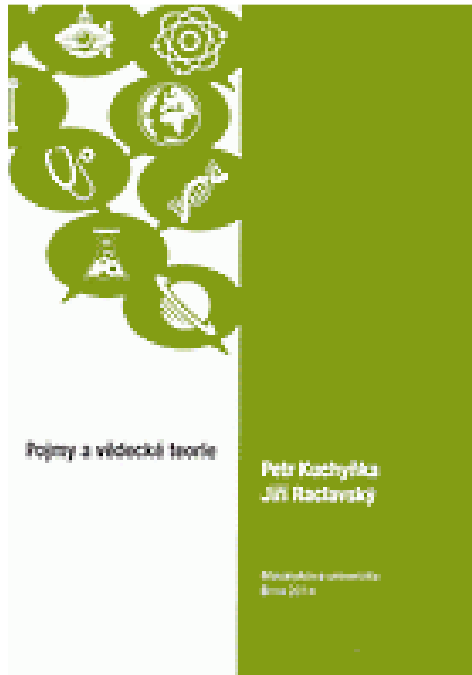


*Transparentní Intenzionální Logika jako
procedurální logika –image missing*

Materna (2004), an application of TIL to the theory of concept and conceptual systems

Duží and Materna (2012), a Czech version of Duží et al. 2010 with some minor corrections

I. Overview: Where to get more information about TIL? (cont.)



Kuchyňka and Raclavský (2014) an explication of the notion of scientific theory and a solution of a number of problem of the philosophy of science where the notion of conceptual/derivation system plays a role

I. Overview: Why TIL seems ‘specific’

- it is an *objectual logic* (similarly as Frege’s and Russell’s logic)
- it concerns abstract logical entities (functions and constructions of functions), rather than a peculiar logical notation
- syntactical matters are not principal: the TIL- λ -terms serve only to depict constructions and the subject of study are constructions

I. Overview: Why to prefer, in logic, semantics rather than syntax (1/2)

- “Turning logic into the study of an artificial language (which nobody speaks) does not strike me as the height of wisdom. A formula of symbolic logic, just like a piece of musical notation, is utterly uninteresting in its own right. Its interest stems exclusively from its ability to represent something other than itself.”

(Tichý 1988, Preface)

- “if the formulas are perspicuous then what they represent cannot be more complex, or more difficult to handle, than the formulas themselves.”

(Tichý 1988, Preface)

I. Overview: Why to prefer, in logic, semantics rather than syntax (2/2)

- “If the term ‘(2.2)-3’ is not diagrammatic of anything, in other words, if the numbers and functions mentioned in the term do not themselves combine into any whole, then the term is the only thing which binds them together. ... The term, the linguistic expression, thus becomes more than a way of *referring* to independently specifiable subject matter: it becomes *constitutive* of it.”

(Tichý 1988, chapter One)

- “The disadvantage of the [non-objectual, syntactical logical] approach ... is obvious: once the entities represented by formulas are lost sight of, they cannot be quantified over. Nor can such quantification be mimicked by quantifying over the corresponding formulas. The enterprise of logic (and mathematics) is thus radically stunted.”

(Tichý 1988, Preface)

I. Overview: Areas in which TIL is useful

- semantics of natural language (explication of meanings)
- (consequently,) application in computer science, e.g. HIT data model
- explication of several philosophical notions (truthlikeness, properties, concepts, conceptual systems, ...)
- a plenty of possible applications that are not already elaborated in TIL or even sketched
- the number of TIL scholars is rather low (P. Materna, M. Duží, B. Jespersen, G. Oddie, P. Cmorej, F. Gahér, P. Kolář, J. Štěpán, P. Kuchyňka, J.R., etc.; not all of them are active - three of them have been deans, one chancellor)

I. Overview: General ambitions of TIL

- “I will argue that the ‘hierarchy of entities’ [Tichý’s type theory] is ... the right medium for modelling our whole conceptual scheme” (Tichý 1988, Preface)
- the main benefit of TIL for a philosophical logician:
 - 1) reasonable philosophical justification of its core
 - = no questionable, strange, or unclear ideas such as dialetheias, etc.
 - 2) clear and straightforward generalizations of the basic ideas
 - = no ad hoc supplementations
 - 3) one extensive logical framework capable to treat a number of phenomena
 - = one needs no particular logics developed for different purposes in particular areas
 - 4) connection to implementation in computers (TIL-formulas *are* λ -formulas, a number of computer languages based on λ -calculus)

I. Overview: Brief history of TIL (1 of 3)

- Tichý's system preceding TIL in 1968 (yet in Czech) and 1969 (*Studia Logica*)
- the first paper on TIL is in *Noûs* 1971; in that version, TIL is Church's simple theory of types with possible worlds + elements of Tichý's semantic doctrine
- a huge unpublished monograph *Introduction to Intensional Logic (IIL)* 1973-76
- late 1970s: various papers which are excerpts from IIL (subjunctive conditionals etc.)
- early 1980s: the papers on deduction also based on IIL
- 1980: Tichý's supreme semantic papers (verb tenses, verb aspects)

I. Overview: Brief history of TIL (2 of 3)

- 1970-90s: papers of various philosophical puzzles (truthlikeness, the ontological proof, theory of references, ability and freedom, ...)
- 1988: his only published book, an excellent analysis of Fregean logical systems, *The Foundations of Frege's Logic* (de Gruyter), exposition of his ramified type theory and many more (e.g. Tichý's final analysis of belief sentences or his philosophy of inference)
- 1990s: unfinished *Meaning Driven Grammar* is a highly ambitious analysis of English
- 2004: posthumously published collected papers

I. Overview: Brief history of TIL (3 of 3)

- a number of scholars in Czech and Slovak republic (but not exclusively: Graham Oddie, Bjørn Jespersen) investigated TIL (most notably: Pavel Materna, Pavel Cmorej, Marie Duží, František Gahér, L. Bielik, J.R., I. Pezlar); several books and hundreds of papers have been published
- Duží, M., Jespersen, B., and Materna, P. 2010 is mostly a codification of some novel applications of TIL especially in semantics
 - *deductive turn*: around 2012
 - (my paper *Je Tichého logika logikou? (O vztahu logické analýzy a dedukce)*, retained by Filozofický časopis from 2010, published in 2012; an English version appeared in Logic and Logical Philosophy, 2014)

II. Introduction to main notions - the apparatus

- functions vs. constructions
- type theory
- deduction

II. Constructions: Two notions of function

1. function as a mere mapping, mere correspondence of arguments and values
 - i.e. functions in the ‘extensional sense’
 - they satisfy the Principle of Extensionality of Functions (PEF)
 - they have no structure
 2. function as a structured recipe, rule, procedure, a particular way how to get from arguments to values,
 - i.e. functions in the ‘intensional sense’
 - they do not satisfy PEF; they have structure
- for instance, Russell in the era of his no-class theory excluded functions in the first sense in favour of functions in the second sense (‘propositional functions’)
 - Tichý treats functions in both sense: the former under the name *functions*, the latter under the name *constructions*

II. Constructions: Objects and their constructions

- constructions are structured abstract, extra-linguistic procedures (‘algorithmic computations’)
- any object O is constructible by infinitely many *equivalent* (more precisely v -congruent, here v is valuation), yet *not identical*, constructions
- each construction C is *specified* by two features:
 - i. which object O (if any) is v -constructed by C
 - ii. how C constructs O (by means of which subconstructions)
- note that constructions are closely connected with objects
- an extensive defence of the notion of construction in (Tichý 1988)

II. Constructions: Kinds of constructions

– for exact specification of constructions see (Tichý 1988)

– four kinds of constructions

(where X is any object or construction and C_i is any construction):

a. variables	x	(not as letters!)
b. trivializations	0X	(‘constants’)
c. compositions	$[C\ C_1...C_n]$	(‘applications’)
d. closures	λxC	(‘ λ -abstractions’)

– definition of subconstructions, free/bound variables

– constructions v -constructing nothing are v -improper

– TIL- λ -terms are used only to denote constructions

– (trivializations of first-order entities, 0X , will be written X)

II. Constructions: Example of equivalent constructions

- for instance, the following function φ :

$$1 \rightarrow -2$$

$$2 \rightarrow 1$$

$$3 \rightarrow 6$$

$$\vdots \quad \vdots$$

- is v -constructed by infinitely many constructions (of various kinds), e.g.:

$$\lambda n \ [[n \times n] - 3]$$

$$\lambda n \ [\ [n + [\text{SquareOf } n]] - [3 + n] \]$$

φ (the trivialization of “babig” directly constructs “babig”)

[IdentityFuntion φ]

f (on a suitable valuation v for the variable v -constructing
unary numerical functions)

II. Type Theory: Simple theory of types (STT, til 1988)

- already in (Tichý 1976)
- let B (basis) be a set of pair-wise disjoint collections of objects:
 - a. every member of B is an (atomic) type over B
 - b. if ξ, ξ_1, \dots, ξ_n are types over B , then $(\xi\xi_1\dots\xi_n)$, i.e. collection of total and partial functions from ξ_1, \dots, ξ_n to ξ , is a (molecular) type over B
- for the analysis of natural discourse let $B_{\text{TIL}} = \{\iota, o, \omega, \tau\}$, where ι collects individuals, o collects truth-values (T and F), ω collects possible worlds (serving as modal index), τ collects real numbers (serving as temporal index)
- functions from ω and τ are intensions (propositions, properties, relations-in-intension, individual offices, ...)

II. Type Theory: Tichý's theory of types (TTT, post 1988)

- in early Tichý's logic, constructions were 'hovering' over STT
- in TTT, constructions occur inside the theory
- cf. precise definition of TTT in Tichý 1988, ch. 5; here only a sketch:
 1. STT given above = i.e. first-order objects
 2. first-(second-, ..., n -)order constructions (members of types $^*_1, ^*_2, \dots, ^*_n$)
= constructions of first-(second-, ..., $n-1$ -) order objects (or constructions)
(i.e. something like a Russellian RTT)
 3. functions from or to constructions
- (Church-like) cumulativity: every k -order construction is also a $k+1$ -order construction
- 'speaking about types' in a bit richer TTT than TTT with B_{TIL}

II. Type theory: Vicious Circle Principles (VCP's)

- in Tichý's ramified theory of types, they are in fact implemented four VCPs (Raclavský 2009)
- VCPs follow the *Principle of Specification*: one cannot sufficiently specify an item by means of the item itself
- *the Functional VCP*: no function contains itself as its own value (or a part of it) or its own argument (or a part of it)
- *the Constructional VCP*: no construction constructs itself or anything of which it is a sub-construction (e.g. variable constructing constructions, *c*, cannot *v*-construct itself - otherwise *c* would not be specifiable)
- *the Constructional-Functional VCP, the Functional-Constructional VCP*

II. Deduction: Tichý' system of deduction

- esp. in (Tichý 1982, which is an extract from Tichý 1976, Tichý 1986)
- only essentials are presented here
- $X:C$ is a *match* M , i.e. that the construction C constructs the/an object O
(X is the trivialization of O , a variable x for objects such as O , or nothing - empty match)
- $\Phi \rightarrow M$ is a *sequent* where Φ contains n matches and M is a match; sequent is *valid* if every valuation satisfying members of Φ satisfy also M
- *derivation rules* R are made from sequents: $\Phi_1 \rightarrow M_1; \Phi_2 \rightarrow M_2; \dots; \Phi_k \rightarrow M_k \models \Phi \rightarrow M$
- corresponding to axiomatic ('reasoning') systems, there are derivation systems (Raclavský & Kuchyňka 2011); $DS = \langle \{C_1, \dots, C_n\}, \{R_1, \dots, R_n\} \rangle$
- derivation rules *exhibit properties of* (and relations between) *objects and their constructions* (Raclavský & Kuchyňka 2011)

II. Summarizing the apparatus

- constructions are the primary target of study, λ -terms are only auxiliary
- the particular version of λ -language was chosen only for the fact that its terms are capable to faithfully depict constructions
- thus there is no need of supplementing these λ -terms by some special semantics (e.g. by inference rules)
- the semantics of ‘TIL language’ is only trivial: the term “ C ” means the construction C
(note that “ C ” does not mean the object O , most frequently a set-theoretical object, constructed by C)
- (setting restrictions by VCPs aside), there are no restrictions on forms of constructions - possibilities to quantify over higher-order objects;
= very (very!) high expressibility of ‘TIL language’

III. Main applications of TIL

III. Explication of meaning

- Tichý himself saw the primary area of application of TIL in the explication of meaning
- the principal contributions were done by Tichý himself
- major part of all TIL research has concerned with the explication of meaning
- in order to explicate an area of intuitive notions one must:
 - specify atomic types (members of B) and objects sorted within atomic types
 - then, all total and partial functions over B , and even constructions of such objects are given (generated by TTT)
 - specify which atomic objects explicate which intuitive primitive notions
 - then, explication of complex intuitive notions is provided (it is 'supervening' on the explication of primitive notions)

III. First fundamental problem in semantics of 20th century

- intensional phenomena inadequately explained by extensional semantics

X believes that the Morning star is the Morning star. (a belief sentence)

The Morning star is the Evening star. (an empirically established identity)

Therefore, X believes the Morning star is the Evening star.

(by Substitutivity of Identity)

X seeks the king of France. (notional attitude report)

Therefore, there is an individual which is the king of France. (by Existent. Import)

- (btw.: Russell's and Russellian analysis of descriptions is wrong!)

III. Solution to the intensional semantic puzzles (1/4)

- to adopt intensions as possible semantic values of expressions
- Richard *Montague* - intensions assigned to expressions only in intensional contexts which is an ad hoc solution
(a circle: context is oblique if existential generalization, etc., does not work correctly; existential generalization, etc., does not work correctly if the context is oblique)
- Tichý - distinction between empirical and non-empirical expressions (below); empirical expressions denote intensions even in *transparent* context

III. Solution to the intensional semantic puzzles (2/4)

- *empirical expressions* - they reference may vary,
- examples: “It rains in Paris”, “the king of France”, “cat”
- they denote *intensions*

- *non-empirical expressions* - they reference cannot vary (in a synchronically given language)
- examples: “seven”, “and”, “every”, “Bill Clinton”
- they denote *non-intensions*

(hyperintensional linguistic phenomena - denotation of constructions; see below)

III. Solution to the intensional semantic puzzles (3/4)

- two kinds of *suppositions* of empirical expressions in expressions
- de dicto/de re suppositions must not be confused with de dicto/ de re beliefs (etc.)
- *de re supposition* - used to deploy the referent; e.g. “the Pope” (referring actually to Ratzinger) in “The Pope is happy”
(note that “the Pope” denotes the intension *POPE* even in this context, denotation is given by linguistic convention)

$\lambda w [\text{Happy}_w \text{Pope}_w]$

- *de dicto supposition* - not used to deploy the referent; e.g. “X seeks the king of France” (referent, if any, is irrelevant)

$\lambda w [\text{Seeks}_w X \text{TKingOfF}]$ (TKingOfF is not applied to w , which blocks

the inferences illustrated in the fundamental problem no. 1)

III. Solution to the intensional semantic puzzles (4/4)

- ambiguity of expressions -> disambiguation on the level of logical analysis (not that the ambiguity of expressions is mimicked by logical ambiguity)
- a very simple idea behind Tichý's theory of de dicto and de re: a function may occur either alone ("sine is a periodical function") or as applied to an argument ("sine 1 is odd"); intension is a function; intension can be, but they need not to be, applied to the (unmentioned) possible world parameter
- the sentence "The Morning star = the Evening star" is about the fact that the two intensions in question (*THE MORNING STAR*, *THE EVENING STAR*) has the same value
- the sentence "X seeks the king of France" is not about an attitude towards an individual but about an attitude towards individual-in-intension (Tichý: *individual office*)

III. Tichý's intensional logic as logic

- classical logical rules are obviously valid only for non-empirical expressions
- in (Tichý 1976), Tichý called them de dicto rules and formulated also de re rules for empirical expressions
- in (Tichý 1976) partial functions (incl. partial intensions) are allowed, so the rules are a bit complicated
- lately Tichý' added the temporal parameter
- Tichý's papers on deduction in early 1980s are compressions of all the work (especially de dicto and de re rules are combined into sophisticated composite rules)

III. Tichý's TIL vs. Montague's intensional logic

- both systems are nearly as old; key differences between the two early versions of the systems are described in (Tichý 1971)
- Montague: empirical expressions denote intensions only in intensional context (contextual shifts of denotation); Tichý: empirical expressions denote intensions in all (even transparent) contexts
- Montague: hat/cup notation; Tichý: easy treatment of modal (w) parameter (λw [... w ...])
- systematic adoption of temporal parameter in the late 1970s by Tichý; Montagovians: in early 1980s
- analysis of temporal discourse (1980), episodic verbs (1980) by Tichý
- Tichý: hyperintensional level of meaning (constructions) already in early 1970s

III. Second fundamental problem in semantics of 20th century

- hyperintensional phenomena inadequately explained by intensional semantics

X believes that $2+3=5$.

$5 = \sqrt{25}$.

Therefore, X believes that $2+3 = \sqrt{25}$.

X believes that it rains in Paris.

$p \leftrightarrow (p \wedge \text{Taut})$

Therefore, X believes that it rains in Paris and [Fermat Last Theorem].

X calculates $2+3$.

Therefore, there is a number (?) calculated by X.

III. Solution to the hyperintensional semantic puzzles

- a number of ‘syntactic approaches’ (Carnap, but even some contemporary writers):
subject-agent is related (at least partly related) to an expression of a natural or formal language denoting an intension/extension
- yet Church’s criticism of Carnap is still valid
- (another proposals – Cresswell, Zalta, Bealer, Thomason – cannot be discussed here)

- a subject-agent has an attitude to the ‘mode of presentation’ of an intension/extension
- Tichý suggests constructions of intensions/extensions as such ‘modes of presentations’
(common criticism of ‘Fregean semantics’ does not hold here, Tichý utilizes also immediated-direct Sinne)

III. Semantic scheme

- in the mid 1970s Tichý suggests the following semantic scheme:

expression E

| E expresses:

construction C = meaning of E

| C constructs, E denotes:

intension/non-intension = denotatum of E

III. Tichý's solution to the hyperintensional semantic puzzles

X calculates 2+3.

$\lambda w \lambda t [\text{Calculates}_{wt} X {}^0[2 + 3]]$ (a construction of a proposition)

- ${}^0[2 + 3]$ constructs directly $[2 + 3]$
- $[2 + 3]$ is not used to construct a number, it is only 'mentioned'
- *X is related just to the construction-procedure* $[2 + 3]$, not to any other construction
- thus one cannot substitute (say) $[\sqrt{25}]$ for $[2 + 3]$; the undesirable consequences are thus blocked
- quite analogously for *belief sentences*, e.g., 'X believes that ...' - subject is *related just to the meaning of the embedded sentence*, that construction
- without explicit treatment of constructions within the system such solution is impossible

III. Topics in semantics treated in TIL

- descriptions (original theory), proper names, predicates, ...
- subjunctive conditionals
- questions
- belief sentences
- modalities
- temporal discourse (tenses, adverbs, ...)
- episodic verbs (verb aspects, ...)

- Introduction to Intensional Logic (finished in 1976) - extensive application of TIL to language
- a number of papers by Tichý (2004)
- some contribution even in (Tichý 1988)

III. Semantics treated in TIL - towards the future

- automatic (computer) logical analysis of Czech language (by A. Horák; Faculty of informatics, MU) based on the aforementioned principles and works by Tichý
- *Meaning Driven Grammar*, unfinished book and project
- sample in “Cracking of the Natural Language Code” (in Tichý 2004)
- very sophisticated approach to natural language meaning
- (the old approach characterized by Tichý as belonging to the era of pencil and paper, to the present era of computers)

III. Some other applications of TIL

- Tichý-Oddie's approach to verisimilitude counting (measuring the distances of theories from truth)
- Tichý's work on deduction with partial functions
- data modelling - HIT model
- Materna's theory of concepts and conceptual systems
- Tichý, Oddie, Cmorej, and J.R.; several applications in metaphysics (esp. study of kinds of properties)
- Oddie and value concepts
- semantical (and other) paradoxes - J.R.
- Kuchyňka's and mine work on derivation systems

Key references

Tichý, P. (1976): *Introduction to Intensional Logic*. Unpublished ms.

Tichý, P. (1988): *The Foundations of Frege's Logic*. Walter de Gruyter.

Tichý, P. (2004): *Pavel Tichý's Collected Papers in Logic and Philosophy*. Svoboda, V., Jespersen, B., Cheyne, C. (eds.), Dunedin: University of Otago Publisher, Prague: Filosofia.

Materna, P. (2004): *Conceptual Systems*. Berlin: Logos.

Duží, M., Jespersen, B., Materna, P. (2010): *Procedural Semantics for Hyperintensional Logic: Foundations and Applications of Transparent Intensional Logic*. Springer Verlag.

Duží, M., Materna, P. (2012): *Transparentní Intenzionální Logika jako procedurální logika*. Bratislava: Aleph.

Raclavský, J. (2009): *Names and Descriptions: Logico-Semantical Investigations* (in Czech). Olomouc: Nakladatelství Olomouc.

Kuchyňka, P., Raclavský, J., (2014): *Concepts and Scientific Theories* (in Czech). Brno: Masarykova univerzita