Partiality

and Transparent Intensional Logic

INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Logika: systémový rámec rozvoje oboru v ČR a koncepce logických propedeutik

pro mezioborová studia (reg. č. CZ.1.07/2.2.00/28.0216, OPVK)

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Abstract

In the historical introduction we discriminate partial logic from three-valued logic. Then, Tichý's adoption of partial functions in his simple type theoretic framework is explained (incl. invalidity of Schönfinkel reduction). When solving a problem with partial function raised by Lepage and Lapierre, we disclose partiality in constructing involved in Tichý's constructions, which is partly based on partiality of functions (mappings) constructed by some subconstructions of those abortive constructions. We show two ways how to overcome partiality: definiteness operator and dummy value technique. Partiality invalidates beta-reduction, which was known already to Tichý; moreover, it invalidates also eta-reduction as I discovered a time ago.

Content

I. Historical part

II. Tichý's logical framework and adoption of partiality in it

III. Work with partiality (definiteness etc.)

IV. Some other work with partiality (conversions)

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I. Historical part

- a brief history of partiality
- philosophical aspects
- some technical issues
- still simple type theoretic framework

I.1 Church's theory of types

- in 1940, Alonzo Church published one of the most known logical system, often called 'typed lambda calculus' or 'simple type theory' (both terms are incorrect for some reasons)
- except the language of lambda calculus and set of deduction rule and axioms, Church's paper involves a definition of (the most known) simple theory of types (here with some little generalization, Church used ο = {T,F}, ι = {I_1, ..., I_n}))

Let α and β are any pairwise disjoint collections of objects:

1. Both α and β are *types*.
2. If α and β are types, then $(\beta\alpha)$ is a *type* of (total) functions from α to β.
3. Nothing other is a *type*.

- most authors write 'α→β' instead of '$(\beta\alpha)$' or even 'B^A'

I.2 Church and Schönfinkel - total functions and reduction

- Church treated only *total* functions of *one* argument
- he presupposed a reduction proposed by Moses Schönfinkel:
  any (total) function of *n*-arguments is known to be representable by a function of 1 argument which leads to the appropriate *n*-1-function:

$$X^{Y \times Z} \approx (X^{Y})^{\times Z}$$

- the reverse of schöfinkelization is known as *currying* (*cf.* its practical use e.g. in investigation of generalized quantifiers), though *un*currying (=schöfinkelization) is also called this name

I.3 Strawson; free logic

- linguistic intuition of Peter F. Strawson led him (1950, On Referring) to the (partly mistaken) criticism of Russell's celebrated analysis of descriptions (1905, On Denoting)
- Strawson said that if a description does not pick out a definite individual, *cf.* "the king of France" or "my children", the sentences such as

  "The king of France is bald"

  are without a truth value, there is (truth-value) *gap* (1964, Logical Theory)
- cf. recent discussion within the theory of singular terms and presuppositions
- technical implementation of the idea of "*non-denoting terms*" (incl. "Pegasus") is by Karel Lambert in 1960s within his *free logic* (*cf.* also some recent revival)

I.4 Semantic paradoxes and truth-valued approaches to it

- already Dimitri A. Bočvar (1938) thought that the liar sentence (L: "L is not true") possess a special truth value, the paradoxical value
- codification of *three-valued* (*3V-*) *logic* in S.C. Kleene's seminal textbook (1952)
- in the discussion of the Liar paradox in the late 1960s (mainly Robert Martin, 1968) there is a disagreement with (Tarski's) bivalence, proposing that L possess no truth-value (since it is 'meaningless', 1980s: 'pathological')
- early 1970s: employing supervaluation technique and (Kleene's) 3V-logic to solve the Liar (Woodruff and Martin, Kripke 1975); such approach was reiterated many times under various guises
- in 1990's, Nuel Belnap's very influential *FOUR* include both truth-value *glut*, but also *gap*

I.5 3V-logic is not 2VP-logic

- common mistake: thinking that 3V-logic is 2VP-logic (2V-logic admitting partiality)
- elementary combinatorics says that there is much *more* total 3V-monadic truth-functions (to have an example) than 2VP-monadic truth-functions
- (attempts to choose only an appropriate amount of 3V-functions as representives of 2VP functions)
- the source of the mistake:

  i. unwillingness to work with partial function directly ('horror vacui')

  ii. confusion of 3V-logic as our language (theory) for a description of 2VP-logic

  (attempts to represent partiality gaps by real objects Undefined, *cf*. e.g. Lapierre 1992, 522; Lepage 1992, 494; hardly philosophically acceptable for the case of, say, individuals)

I.6 Summing up motivation for admitting partiality (gaps)

- linguistic/semantic intuitions (Strawson and the theory of singular terms)
- ontological assumptions (not only existence issues, even a belief in gaps in reality, *cf*., Imre Ruzsa 1991)
- technical convenience in the case of troubles such as the Liar (Kripke etc.)

more persuading reasons:

- *computer scientists* seem to generally agree that partiality is *needed for an adequate description of programme behaviour* (*cf*. the corresponding literature)
- *general logical reason* that we should treat not only total functions when *partial functions cannot be reduced to the total ones* (*cf*. below)

I.7 Irreducibility of partiality (cont.)

- not only in natural languages, even in mathematical discourse partiality is exhibited (“3÷0”); i.e. foundational issues - without clarifying the issues of partial functions we cannot clarify which semantical value is possessed by a problematic expressions such as gappy sentences etc. (cf. e.g. Feferman 1995)

- total functions are only special cases of partial functions (Lepage 1992, 493); which is in fact a *‘lack of generality’ argument*

- some authors distinguish partial, total and nontotal functions (Lapierre 1992, 517)
- following Tichý, *partial function* is a function (as a mapping) which is not defined for at least one argument; *total function* is a function defined for all arguments

I.8 Tichý's adoption of partiality

- in 1971 (Studia Logica), Tichý published an elegant modification of Church's (1940) to be applicable to natural language semantic analysis; there, partiality is not discussed at all
- issues within the realm of semantic analysis which were discussed in that time seem to be the most probable reasons why Tichý faced partiality
- between 1973-76 Tichý wrote an extensive monograph Introduction to Intensional Logic
- there, partiality is systematically adopted in the type theory and the deduction system for it
- a compressed selection for this book was published in *1982* in Reports on Mathematical Logic as "*Foundations of Partial Type Theory*"
- this paper is generally recognized as the *first paper introducing partial function into the type theory*

I.9 Tichý's 'anti-schönfinkelization' argument

- main complication with partial functions*: Schönfinkel's reduction does not work* (Tichý 1982, 59-60)
- let us have a partial binary truth-function *f* (/ (ooo)) defined:

$$f(x,y) \begin{cases} y \text{ if } x=0 \\ \text{undefined otherwise.} \end{cases}$$

(i.e. yields nothing for ⟨1,0⟩ and ⟨1,1⟩)

- 2 total (o(oo))-function correspond to *f*; both maps 0 to the identity truth-function; but the first is undefined for 1, while the second maps 1 to the monadic truth-function undefined for all values
- in other words, the work with total *monadic* functions cannot replace work with partial functions (not: total functions cannot replace work with partial functions)

I.10 Tichý's simple theory of types with partiality

- in Tichý (1982, 60) (and already 1976), Tichý proposes a generalized form of Church's definition of simple type theory (STT)
- it has open basis (even the number of truth-values is unsettled) and accepts polyadic partial functions

Let *B* (basis) consist of mutually non-overlapping collections of objects.

a. Any member of *B* is a *type over B*.

b. If ζ, ξ_1, ..., ξ_m are (not necessarily distinct) types over *B*, then $(\zeta\xi_1...\xi_m)$, which is a collection of all total and partial functions from ξ_1, ..., ξ_m into ζ, is a *type over B*.

c. nothing is a *type over B* unless it follows from a-b.

- (Tichý never made an effort to define partial functions as some other objects, as some authors do, *cf.* e.g. Lapierre 1992, Lepage 1992)

II. Tichý's logical framework and adoption of partiality in it

- constructions
- ramified theory of types
- resolving some small issues

II.1 Two kinds of functions

- historical development of the notion of function
- function-as-mapping, function-as-rule (procedure)
- from the first half of 1970s Tichý sharply discriminate between the two:
    a. *functions* (mere mappings)
    b. *constructions* (not confuse with the intuitionistic notion)

- functions are individuated as satisfying (modified) extensionality principle (Raclavský 2007)
- but constructions have intensional individuation: they can be *equivalent* but *not identical*
- defence of the notion especially in Tichý (1988, 1986)

II.2 Constructions (v-)construct objects

- constructions are abstract (extralinguistic) objects akin to algorithm (algoritmic computation, but not necessarily effective)
- dependently on valuation v, construction v-construct objects
- any object O (even a construction!) is v-constructed by infinitely many constructions
- constructions are 'ways', 'procedures' how to arrive to, obtain, an object
- any construction C is given by:

  i. the object (if any) v-constructed by C

  ii. the way how C v-construct it (by means of which subconstructions

II.2 Constructions (v-)construct objects (cont.)

(cf. modes of forming constructions)

- constructions are often *denoted* by Tichý's lambda-terms (objectual, procedural lambda-calculus)
- *v-congruence* of C and $D =_{df}$ the two constructions C and D v-construct the same object (or no object at all)
- *equivalence* of C and $D =_{df}$ for all v, C is v-congruent with D
- C is v-improper $=_{df}$ C v-constructs nothing at all

II.3 Modes of forming constructions

- again, following Tichý (1988, chapter 5)
- valuation is a field consisting of sequences, each sequence being a total function to objects of a unique type

i. *variable* x_k; it v-construct the kth-entity from v of the appropriate type

ii. *trivialization* ^0X (X is any object or construction); for any v, v-constructs X without help of any other constructions

iii. *composition* $[C\ C_1 \ldots C_n]$; applies the entity (if any) v-constructed by C to the string of entities (if any) v-constructed by $C_1, \ldots, C_n$

(tbc.)

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II.3 Modes of forming constructions (cont.)

iv. *single execution* ^1X is here omitted

v. *double execution* ^2X; ^2X *v*-constructs the entity (if any) *v*-constructed by X (if X is a construction)

vi. *closure* $\lambda x\ C$; for any v, $\lambda x\ C$ *v*-constructs function from entities in the range of x to the entities (appropriately - simplified formulation) *v*-constructed by C

II.4 Semantic theory - Transparent intensional logic

- not confuse Tichý's logic with its particular instance known as Transparent intensional logic (*TIL*)
- TIL treats not only extensional objects but also possible world intensions (= some mappings)
- B_{TIL}={ο,ι,ω,τ} (i.e. 2 truth-values, individuals, possible worlds, real numbers)
- intensions are treated explicitly (λ*w*λ*t* [...*w*...*t*...] whereas *w*/ω, *t*/τ; "/" abbreviates "*v*-constructs an object of type")
- the non-logical part of TIL is a set of semantics doctrines (who to analyse natural language expressions); the main is that expressions *express* constructions and *denote* the object constructed by the constructions; empirical expressions ("the King of France", "It rains in Nice") denote intensions
- for semantical applications of TIL see mainly Tichý (1988, 2004), Raclavský (2009), Duží et. al (2010)

II.5 Tichý's ramified theory of types

- see the definition in Tichý (1988, ch. 5)
- constructions receive a special type over *B*
- constructions are built in a non-circular manner; thus there is in fact a lot of types of constructions which differs in their *order* (it is thus RTT): $*_1$, $*_2$, ..., $*_n$
- functions from or to constructions are also classified, thus the framework is very, very rich
- for discussion of Tichý's RTT *cf.* the works of the present author (e.g. 2009)

II.6 Tichý's system of deduction

- already in Tichý (1976), then esp. in (1982, 1986); not updated to his RTT
- basic idea: work with constrictions, not with expressions
- key items are matches, e.g. x:C ("C v-constructs and object x")
- partiality carefully treated
- problematic rules (e.g. ExImport, ExGeneralization, Substitution) are rectified (the results are unfortunately very, very complicated)
- see e.g. Raclavský (2010, 2012) or explanation of the connection with the realm of entities

II.7 A trouble concerning iterated partial functions

- e.g. Lapierre (1992, 520), Lepage (1992, 494): suppose the function *f* is undefined for the object O, (*f* O) is thus undefined; what is (*f* (*f* O)) ?
  (explanation: (*f* O) has no value; to be a function at all, (*f* (*f* O)) needs to have at least argument)
- but the authors confuse two kinds of functions
- Tichý has two kinds of "partiality":

1. partiality of functions (mappings) and
2. partiality of constructions (v-improperness)

- consequently, the construction $[{}^0f\,[{}^0f\ {}^0\mathrm{O}]]$ is a well-defined construction which applies *f* (*v*-constructed by the first occurrence of ^0f) to the object (if there is any) *v*-constructed by $[{}^0f\ {}^0\mathrm{O}]$; since $[{}^0f\ {}^0\mathrm{O}]$ is *v*-improper, $[{}^0f\,[{}^0f\ {}^0\mathrm{O}]]$ is also *v*-improper

II.8 The type-theoretic freedom of composition

- unlike early Tichý (and Materna, etc.), Tichý 1988 (and also Raclavský 2009+) leaves *composition type-theoretically 'free':* composition of any C with any string of $C_1, ..., C_n$ (type theoretic unfreedom: C must be a construction of a function of a type consonant with the types of $C_1, ..., C_n$)
- this opens doors for many 'partiality-gapiness' phenomena
- a practical example in which the freedom is useful (Raclavský 2008), "Xenie thinks that Ceasar is a prime number" (note the type-theoretic mismatch category mistake); by compositionality, "Ceasar is a prime number' *also* has a meaning; one can modelled it only with the type-theoretic freedom of composition

III. Working with partiality

- definiteness operator
- dummy value technique

III.1 Definiteness operator - *unfinished slides*

- a well-known idea: *d*!, where "*d*" is a term, yields a definite value despite "*d*" is an empty term
- in TIL, several such operators are definable
- for instance *C* *v*-constructs an *o*-objects, but it is *v*-improper for some *v*; ! /(ο$*_k$) (maps construction to definite truth-values); o/ο; T/ο (True);

$$[{}^0! \, {}^0C] \quad \Leftrightarrow \quad [{}^0\exists \lambda o \, [\, [o \; {}^0\!= C] \; {}^0\!\wedge [o \; {}^0\!= {}^0\mathrm{T}] \,]$$

- another one, called "True$^{\pi\mathrm{T}}$" (p/ο$_{\omega\tau}$; w/ω; t/τ; Raclavský 2008):

$$[{}^0\mathrm{True}^{\pi\mathrm{T}}{}_{wt} \, p] \quad \Leftrightarrow \quad [{}^0\exists \, \lambda o \, [\, [o \; {}^0\!= p_{wt}] \; {}^0\!\wedge [o \; {}^0\!= {}^0\mathrm{T}] \,]]$$

III.2 Dummy-value technique

- for some purposes we need some 'dummy' (or 'null') value if an application of a function is unsuccessful
- published in Raclavský (2010)
- imagine that we count salaries of individuals and certain individual allegedly having a salary is described as 'the king of France'; '...+the salary of (KF)+...'; to avoid whole sum being undefined, we want the salary of (KF) to return (say) 0 i.e. the dummy value

III.2 Dummy-value technique (cont.)

- let Sng be a type-theoretic appropriate singularization ('ι-operator'); e.g.: $x,y,z/\iota$; $f/(\tau\iota)$; ^{0}T/o; If_Then_Else is the familiar ternary truth function:

[^{0}Sng.λz [^{0}If_ [${}^0\exists.\lambda y$ [y ^{0}= [$f\, x$]]]

checking whether f is defined for x

Then [${}^0\exists.\lambda o$ [[o ^{0}= [z ^{0}= [$f\, x$]]] ${}^0\wedge$ [o ^{0}= ^{0}T]]]

checking whether z is value of f

_Else [z ^{0}= ^{0}DummyValue]]]

slipping our dummy value (of type ι)

- note carefully that the function f is still without a value for 'bad' argument; we only 'repair' this because we want the construction in which f occurs not to be v-improper

IV. Troubles with reductions

- trouble with eta-reduction

- troubles with beta-reduction(s)

IV.1 Conversions

- Church used conversion *lambda-rules* I.-III.

- these were lately modified by Curry to *alfa*-, *beta*- and *eta*-conversions

- (*conversion* = *reduction* or *expansion*)

- for more see esp. Raclavský (2009)

IV.2 Classical beta-reduction

- β-reduction is the basic computational rule of λ-calculus; it preserves *v*-congruency of terms/constructions
- let us abstract from the problems with appropriate renaming of variables
- within frameworks using *total functions only*, the conversion rules is exposed as (the left formula is often called β-*redex*)

$$[\lambda x\, [...x...]\, C] \Leftarrow\!\Rightarrow [...C...]$$

- the method of distinct strategies of reduction is well known and studied (whym because β-redexes can occur in other β-redexes and we ask which way of reducing is quicklier and whether they reach the same result; the questions of termination and normalizing)

IV.3 'Conditionalized' beta-reduction

- in (Tichý 1982, 67), β-reduction and β-expansion are exposed, quite naturally, as deduction rules (The Rule of Contraction, The Rule of Expansion)
- *Tichý's rule of β-reduction contains an explicit condition that the substituted construction C v-constructs something (is not v-improper)*
- without reference to Tichý, several authors (e.g. Beeson 1985, 2004) do the same
- so conditioned, β-reduction preserves *v*-congruence (equivalence) of constructions
- Tichý does not explain *why* he conditionalized β-reduction, *cf.* next slide

IV.4 Call-by-name / call by value

- in explaining β-reduction theoreticians often say that the reduced term is $D'(C/x)$ whereas D is the body of λx [...*x*...] (i.e. it is [...*x*...]) and D' is D in which x is replaced by C and the whole D' is *executed*
- two readings (traditional terminology, Plotkin 1975):

i. *(call-)by name*: C is inserted in D *as it is* and any *execution* goes only after that

ii. *(call-)by value:* C is *executed*, and the result of executing is inserted instead of x in D and such D' is *executed*

- since many theoreticians generally do not discriminate carefully between constructions and functions, such explanation is usually not given and, sometimes, the authors provide their own new distinctions (and prescriptions)

IV.5 Invalidity of beta-reduction 'by name'

- using example by Duží (2003, cf. also Duží et al. 2010, 268-9), *invalidity of β-reduction 'by name'* in the framework adopting partial functions:

D: $\lambda w \lambda t\, [\, \lambda x\, [{}^0\mathrm{Believe}_{wt}\, {}^0\mathrm{Xenia}\, \lambda w \lambda t\, [{}^0\mathrm{Bald}_{wt}\, x]]\, {}^0\mathrm{KF}_{wt}]$

("The King of France is such that it is believed by Xenia to be bald")

E: $\lambda w \lambda t\, [{}^0\mathrm{Believe}_{wt}\, {}^0\mathrm{Xenia}\, \lambda w \lambda t\, [{}^0\mathrm{Bald}_{wt}\, {}^0\mathrm{KF}_{wt}]]$

("Xenia believes that the King of France is bald")

- the construction D constructs a *gappy proposition* because ${}^0\mathrm{KF}_{wt}$ is v-improper
- however, E, the result of inserting ${}^0\mathrm{KF}_{wt}$ *as it is* instead of x in the body of D, constructs a *non-gappy proposition*
- thus, D and E are not *equivalent* (there is no problem with β-expansion 'by name')

IV.6 Beta-reduction ‘by-value’

- Duží et al. (2010) do not mention Tichý’s ‘conditioned’ β-reduction
- they suggest, however, a novel TIL-*β-reduction, ‘by value*’ (p. 269-70); adapting and simplifying it (C, [...x...] / $*_k$):

$$[\lambda x\, [...x...]\; C] \Leftrightarrow {}^2[\, {}^0\mathrm{Sub}^k\, [{}^0\mathrm{Triv}^k\;\; C]\; {}^0x\; {}^0[...x...]]$$

- the right-side construction is an *exact* correlate of ‘by-value’-reading of $D'(C/x)$, a transcription in TIL-language

IV.6 Beta-reduction 'by-value' (cont.)

$[\lambda x\,[...x...]\,C] \Leftarrow\Rightarrow\ {}^{2}[{}^{0}\mathrm{Sub}^{k}\,[{}^{0}\mathrm{Triv}^{k}\ C]\ {}^{0}x\ {}^{0}[...x...]]$

- the right side construction is of an excellent construction-form developed by Tichý:
- ${}^{0}\mathrm{Triv}^{k}/(*_{k})\xi$); the function Triv^{k} maps ξ-object (v-constructed by C) to its trivialization
- ${}^{0}\mathrm{Sub}^{k}$ / $(*_{k}*_{k}*_{k}*_{k})$; the (partial) function which puts the A instead of (all directly contained occurences of) B in C (here: [...x...]), yielding thus D; A, ..., D are k-order constructions;
- Tichý's function Sub^{k} preserves v-congruence (!) of input and output constructions, which is very welcome; but this is not surprising because beta-reduction is based is substation technique

IV.6 Beta-reduction ‘by value’ (cont.,cont.)

- using our example:

$^{2}[^{0}\mathrm{Sub}^{1}\ [^{0}\mathrm{Triv}^{1}\ ^{0}\mathrm{KF}_{wt}]\ ^{0}x\ ^{0}\ \lambda w\lambda t\ [^{0}\mathrm{Believe}_{wt}\ ^{0}\mathrm{Xenia}\ \lambda w\lambda t\ [^{0}\mathrm{Bald}_{wt}\ x]]]$

is a construction which is *v*-improper (which we need) because $[^{0}\mathrm{Triv}^{1}\ ^{0}\mathrm{KF}_{wt}]$ is *v*-improper

- but consider also, that $^{0}\mathrm{KF}_{wt}$ *v*-constructs Yannis; then, Yannis is trivialized to $^{0}\mathrm{Yannis}$ and $^{0}\mathrm{Yannis}$ replaces x in $\lambda w\lambda t\ [^{0}\mathrm{Believe}_{wt}\ ^{0}\mathrm{Xenia}\ \lambda w\lambda t\ [^{0}\mathrm{Bald}_{wt}\ x]$
- important observation: an ideal β-reduction ‘by value’ (not yet defined), which is sensitive to partial gapiness, involves all cases captured by Tichý’s ‘conditionalized’ β-reduction; it would be thus better than Tichý’s proposal (but *cf.* below)
- a remark: within his pre-1988 framework, Tichý has not possibility to define β-reduction ‘by value’

IV.7 Beta-reduction 'by value' - the problem with too high order

- important observation, book Duží et al. cannot provide a sufficient framework for 1st- order and 2nd-order deduction
- generally, if we investigate *k*-order deduction system, we can enrich by an explicit (but simple form) of β-reduction rule only its *k*+2 order correlate (*metameta*!)
- because even in the simplest case, the lowest order of (explicit) β-reduction 'by value' is 3

  (let the order of *D*, etc., is *k*; ^{0} applied to *D* increases the order of the definiens from *k* to *k*+1; ^{2} also increases the order, the order of $^2[\ldots\ ^0D\ ]$ is thus *k*+3)
- a way out seems to be (only *metalevel*), but the problem is still studied:

$$[^0\text{Beta-Reduced}\ ^0C\ ] \Leftrightarrow\ ^2[\ ^0\text{Sub}^k\ [^0\text{Triv}^k\ C]\ ^0x\ ^0[\ldots x\ldots]]$$

IV.9 Beta-reduction 'by value' - a note on the 2010-definition (cont.)

- the whole (yet still simplified) form of Duží's β-reduction 'by value' which try put items of the string C_1C_2 inside $[...x_1...x_2...]$ contains an error

$$[\lambda x_1 x_2\, [...x_1...x_2...]\, C_1C_2] \Leftrightarrow\ {}^2[\,{}^0\mathrm{Sub}^k\, [{}^0\mathrm{Triv}^k\ C_1]\, {}^0x_1\ {}^{0!}[\,{}^0\mathrm{Sub}^k\, [{}^0\mathrm{Triv}^k\ C_2]\, {}^0x_2\ {}^0[...x_2...]]\,]$$

- $^{0!}$ ('!' only helps to indicate which '^{0}' is discussed) must be deleted unless the definiens is ill-formed
- reason: let the result of C_2 is X_2, then the result of $[{}^0\mathrm{Sub}^k\, [{}^0\mathrm{Triv}^k\ C_2]\, {}^0x_2\ {}^0[...x_2...]]$ is $[...\mathrm{X}_2...]$ in which we may substitute (which we wish); Duží's $^{0!}[...\mathrm{X}_2...]$ is not open tu substitution, x_1 in $^{0!}[...\mathrm{X}_2...]$ is not hospitable for the result of $[{}^0\mathrm{Triv}^k\ C_1]$

IV.10 Failure of classical eta-reduction

- classical η-conversion rule is said to expresses extensionality of ‘functions’
- where C stands for a function-mapping:

$$\lambda x\,(C\,x) \Leftrightarrow C$$

- even in type-theoretically unfree composition approach, η-*reduction is not generally valid* (Raclavský 2009: 283, 2010: 126)
- in (2009, 2010) I have suggested to ‘conditionalize’ the η-reduction rule

IV.10 Failure of classical eta-reduction (cont.)

- let x/α; y/β; $F/((\gamma\beta)\alpha)$; X is an α-object;
  note that both $\lambda y\ [[Fx]\ y]$ and $[Fx]$ / $(\gamma\beta)$;
  let the function *v*-constructed by F is undefined for X an x *v*-constructs X;
- then,

$[Fx]$	*v*-constructs nothing (no $(\gamma\beta)$-object), i.e. it is *v*-improper
$\lambda y\ [[Fx]\ y]$	*v*-constructs an $(\gamma\beta)$-object, viz. a function undefined for each its argument (since $[Fx]$ is *v*-improper, $[[Fx]\ y]$ is also *v*-improper)

V. Conclusions

V. Conclusions

- partiality brings various problems
- the problems can be overcome
- there is still a lot work to do
- the power of frameworks adopting partiality can then be utilized

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