
Derivation Systems and Verisimilitude

(An Application of Transparent Intensional Logic)



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Logika: systémový rámec rozvoje oboru v ČR a koncepce logických propedeutik
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Abstract

In the second half of 1970s, a variety of approaches, the first one proposed by Pavel Tichý, defined verisimilitude - likeness of theories to truth - in the framework of intensional logic. Popper's collaborator David Miller objected to Tichý's method that it is not translation invariant because the verisimilitude of a theory is changed after its translation. Tichý and Oddie rightly noticed an ambiguity in Miller's argument. But a proper solution to the problem can be given only if derivation systems are utilized in explanation. The notion of derivation system is defined in Transparent intensional logic. We show that verisimilitude is dependent on derivation systems. The crucial observation is that there are two kinds of simple concepts, primary and derivative ones, while such their feature is based on their position in a particular derivation system.

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I. Introduction

- verisimilitude of theories

I.1 Introduction: verisimilitude (= truthlikeness) of theories

- Popper's falsification of (scientific) theories seems to be a kind of 'negative programme'
- in (1963), Popper suggested a 'positive programme': some theories are *closer to the truth* than others, thus they are better, i.e. a positive progress exists
- theories can be *ordered* with regards to their *verisimilitude* (the term is often replaced by a more accurate term '*truthlikeness*')
- Popper suggested 2 methods of counting verisimilitude: quantitative and qualitative approach
- (see Graham Oddie's (2007) entry in Stanford Encyclopaedia of Philosophy for an overview of the topic)

I.2 Introduction: Popper's verisimilitude counting refuted

- in 1973, Popper visited New Zealand; the point of his lecture was demolished by the Czech logician and philosopher *Pavel Tichý* (1936 Brno -1994 Dunedin), who immigrated to NZ in early 1970s
- Tichý published his criticism in (1974), in The British J. for the Ph. of Sc.
- an analogous criticism was published, in the very same volume of the journal, by *David Miller*, Popper's close collaborator
- at the very end of his 1974-paper, Tichý sketched a *novel method* of a verisimilitude counting based on Hintikka's normal distributive forms; he used there his own weather example (1966; in Czech)
- in his 1974-paper, Miller sketches a criticism of Tichý's method

I.3 Introduction: development of a discussion (1/2)

- in the second half of 1970s, Tichý published two papers (1976, 1978), where the details of the method are exposed and intuitive examples are discussed; moreover, he defends the method against criticism by Popper (who even dismissed the very notion – because of problems), Miller and Niiniluoto
- Miller published several papers in which the method was criticized; he tried to reconcile the notion with the rest of Popperian doctrines
- *note*: Tichý (and also me) is not a philosopher of science, thus he stands a bit outside of the philosophy of science and its internal discussion; this has a negative as well as positive feature (e.g. he never committed to syntactic or semantic conception of theories; the first one utilizes axiomatic method, the latter one utilizes theory of models)

I.4 Introduction: development of a discussion (2/2)

- Tichý's method is framed in *intensional logic* (*possible world semantics*), his former pupil *Graham Oddie* elaborated the proposal in his 1986 book; Oddie reports even existence of a computer program counting verisimilitude (!)
- in 1976, Risto Hilpinen published a counting of distance between possible worlds; roughly, a method similar to Tichý's (in fact: no - see Oddie 2007 for explanation); *Ilka Niiniluoto* developed this approach (a book in 1987), which differs only in minor details (which I am not interesting in) from Tichý's
- a number of other approaches have been suggested (see, e.g., Kuipers 1987); most of them reacts to Miller's language dependence problem
- (I am not going to compare any of these approaches, I am stick to Tichý-Oddie approach)

II. Tichý's method and Miller's argument

- Tichý's method of verisimilitude counting
- Miller's language-dependence argument

II.1 Verisimilitude counting: an example (1/2)

- a simple Tichý's example
- 3 (atomic) *states of world*: *h* (hot), *r* (rainy) and *w* (windy)
- let so-called *truth* (in fact, it is a null theory) be

$$T_0: h \& r \& w$$

- now let the measured theories be:

$$T_1: \sim h \& r \& w$$

$$T_2: \sim h \& \sim r \& \sim w$$

- intuitively, T_2 is worse than T_1 , because it is wrong on more points; another theory, $\sim h \& r$ is worse than T_1 because it is right on less points (but it is better than T_2); the example can be generalized to predicate logic

II.2 Verisimilitude counting - an example (1/2)

- Tichý's verisimilitude counting employs the intuitions we have:

$$\text{ver}(T_k, T_0) = 1 - (\text{number of wrong guesses} / \text{number of guesses})$$

- (btw. $\text{distance}(T_k, T_0) = \text{ver}(T_k, T_0)^{-1}$)

- in our example: $\text{ver}(T_1, T_0) = 1 - 1/3 = 0,66 > \text{ver}(T_2, T_0) = 1 - 3/3 = 0$

- in our example: $\text{ver}(\sim h \& r, T_0) = 1 - 1/2$; i.e. $\text{ver}(T_1, T_0) > \text{ver}(\sim h \& r, T_0) > \text{ver}(T_2, T_0)$

- *note*: further details of the method do not concern us

II.3 Miller's language-dependence argument (1/2)

- Miller (1974, 1976, ...) introduces a novel set of world-states: *h* (hot), *m* (minnesotan) and *a* (arizonan)
- '*translation rules*', definitions:

$$m =_{df} (h \equiv r)$$

$$a =_{df} (h \equiv w)$$

- according to the translation rules and elementary logic, $(h \& r \& w) \equiv (h \& m \& a)$
- thus T_0 (my later notation: T'_0) is now translated as $h \& m \& a$

II.4 Miller's language-dependence argument (2/2)

- after translation of T_1 and T_2 we get (in fact)

$T'_1: \sim h \& \sim m \& \sim a,$

$T'_2: \sim h \& m \& a.$

- since T_0 is $h \& m \& a$, $\text{ver}(T_1, T_0) = 1 - 3/3 = 0$ and $\text{ver}(T_2, T_0) = 1 - 1/3 = 0,66$, i.e. we receive *reverse numeric values for one and the 'same' theory*

before translation: $\text{ver}(T_1, T_0) > \text{ver}(T_2, T_0)$

after translation: $\text{ver}(T_1, T_0) < \text{ver}(T_2, T_0)$

- *Miller's conclusion*: Tichý's method is *inadequate* because the verisimilitude ascertained is *dependent on the linguistic formulation of the theory*; but we demand a *translation invariant method*

II.5 Solutions to Miller's language-dependence problem

- *two nonstarters*:
- the two (system of) theory (-ies) are not comparable, similarly as Newton's and Aristotle's physics; *no*: we *presuppose comparability* in our case
- Miller's novel predicates/notions/states of world should be ruled out as artificial and unusual; *no*: there is *no privileged* conceptual scheme / language / predicate
- because of lack of space, a number of *nontrivial approaches* by various authors are not reported here and will be ignored
- moreover, I will suppress any details of Tichý's (1976, 1978) and Oddie's (a whole chapter in his 1987) analysis; my explanation (2007, 2008, 2008a) is a bit analogous (and is the only right one😊)

III. Logical framework

- what a theory is
- from (so-called) constructions to (so-called) derivation systems

III.1 What is a theory (that can be measured)?

- the crux of the problem: with help of which *kind of model* of scientific theory one should count verisimilitude?
- the paradigm of philosophy of science: syntactic models were rejected in favour of semantic models (etc.); but such models need not to be suitable for verisimilitude counting at all
- as far as I know, an attempt to discuss these matter is present in Oddie (1987, 2007)
- Tichý-Oddie method is to the large extent tied to possibilities of Tichý's sophisticated logical framework; since the framework is so huge, it is difficult to find an alternative answer outside the framework
- *note*: my resolution of Miller's puzzle is based on distinctions of this section

III.2 Tichý's framework - Transparent intensional logic (1/4)

- Tichý's logical framework can be best introduced as an objectually understood λ -calculus over a special ramified theory of types
- recall that λ -calculus is richer than any language based on notation of predicate logic
- in Tichý's papers, a simpler version of his late framework is used
- Oddie tried to hide most of the features of the framework (perhaps to avoid conflicts)
- so it seems that Tichý-Oddie method utilizes only a *notation of λ -calculus*
- for my resolution of Miller's puzzle, I must *introduce the objectual level* (the level of constructions)

III.3 Tichý's framework and explication of theories (1/3)

- within the adopted logical framework, several explications of theories are possible
- each possibility I accept incorporates the idea that a theory is something true or false; thus theory is something like a sentence or a class of sentences
- I distinguish only three basic possibilities:
 1. a theory is a *syntactic object*: a sentence or a set of sentences
 2. a theory is a *semantic set-theoretic object*: a possible world proposition (or a class of propositions) or some other such set-theoretic object
 3. a theory is a *semantic hyperintensional object*: a (Tichý's) propositional construction (or a class of these)

III.4 Tichý's framework and explication of theories (2/3)

- the *option 2.* (propositions) is a bad one: counting of verisimilitude is based on *structural similarity* (Hintikka's forms etc.); possible world propositions *do not have such structure*, thus a method of counting does not match intuitive desiderata (see Oddie 2007 for details)
- the *option 1.* (sentences) is also a bad one: though sentences do have a structure and counting is usually explained as based on sentences of some formal language, sentences are *linguistic objects*; a verisimilitude for a one linguistic embodiment of a theory would differ from verisimilitude for another linguistic embodiment of a theory - well, these are two different theories, not one theory in two shapes!

III.5 Tichý's framework and explication of theories (3/3)

- the *options* 3. is the only viable one: Tichý's constructions are *structured* entities, thus capable to bear *structural similarity*; they can be expressed by sentences of distinct languages, so they are *language independent*
- the only problem is that a. they are not well known in the current paradigm, so they seem rather exotic, and that b. they can be easily confused with their linguistic representations (which seems to be a source of Miller's under-appreciation of Tichý-Oddie method)

III.6 Tichý's framework - Transparent intensional logic (2/4)

- two senses of functions:
 - a. functions as mere *mappings* (correspondence of argument and values)
 - b. functions as *structured* 'recipes', *procedures*, ways how to reach an object
- many logical frameworks utilize only a.-functions; if b.-functions are used at all, they are unrecognized; in many frameworks, b.-functions are replaced by their linguistic representations, e.g. λ -terms
- Tichý's framework utilizes both kinds of functions, the b.-functions are certain *constructions* (*Cs*)
- every object *O* is constructed by *n* (*equivalent*, but *non-identical*) *Cs*
- *Cs* are **extralinguistic** abstract entities of an algorithmic nature
- each *C* is specified by i. which *O* it constructs, ii the way it constructs *O*

III.7 Tichý's framework - Transparent intensional logic (3/4)

- *semantic scheme*:

“It is hot and not rainy” - an expression E (a linguistic embodiment of theory)

| E expresses in L

$\lambda w [H_w \wedge \neg R_w]$ - the construction C of a proposition

| E denotes in L , C constructs

the proposition that it is hot and not rainy

- both the (propositional) construction and the propositions are *language independent*
- *EXPRESSES* and *DENOTES* are language dependent relations, *CONSTRUCTS* is not

III.8 Tichý's framework - Transparent intensional logic (4/4)

- *the theory is* $\lambda w [H_w \wedge \neg R_w]$; in L, it can be expressed by “It is hot and not rainy” and in L^{Czech} by “Je horko, ale ne deštivo” – the two sentences are *translatable* = they express, in L and L', the very same C
- the theory is not expressible in L by an un-isomorphic sentence ‘Not that, if is not hot, then it rains’; the two expressions are only *equivalent* (in L) = they are not identical but they denote (not express!) one and the same proposition
- two constructions are *equivalent* (not: in L) iff they construct one and the same proposition (object...)
- in the debate with me, Miller (and Taliga) showed a very relaxed attitude to the difference between translatability and equivalence, which is unfortunate

III.9 Tichý's framework – derivation systems (1/2)

- *derivation system* (Raclavský, Kuchyňka 2011) is (roughly) a couple

$\langle Cs, DRs \rangle$,

where Cs is a class of constructions and DRs is a class of derivation rules operating on Cs

- *derivation rules* of Tichý's system of deduction (Tichý 1982, 1986)
- *definitions* are certain \Leftrightarrow -rules (Raclavský 2009, 2011)
- derivation systems displays properties of objects constructed by Cs
- note the difference from syntactic entities called 'axiomatic systems'
- some simple Cs are primary, some derived (see below)

IV. Derivation systems and verisimilitude

- verisimilitude and derivation systems
- rethinking Miller's puzzle

IV.1 Derivation systems and verisimilitude

- definitions show which *simple* constructions ('concepts') are *primary* and which are *derived*
- examples (I omit Cs of logical connectives):

$DS^{HRW} = \langle \{H, R, W\}, \emptyset \rangle$, i.e. 3 primary simple Cs, but no derived primary Cs

$DS^{HRW(M)} = \langle \{H, R, W\}, \{M_{wt} \Leftrightarrow_{df} [H_{wt} \equiv R_{wt}]\} \rangle$, i.e. 3 primary simple Cs, and one derived primary C, viz. M

$DS^{HMA(R)} = \langle \{H, M, A\}, \{R_{wt} \Leftrightarrow_{df} [H_{wt} \equiv M_{wt}]\} \rangle$, i.e. 3 primary simple Cs, and one derived primary C, viz. R

IV.2 Derivation systems, verisimilitude and conversion of theories

- the main result of Raclavský (2007) was not anticipated, but only presupposed, by Tichý and Oddie:
- *before counting verisimilitude, each theory has to be converted to employ only primary Cs of the derivation system of T_0*
- example: if T_0 is (say)

$$\lambda w \lambda t [H_{wt} \wedge R_{wt} \wedge W_{wt}],$$

we convert

$$\lambda w \lambda t [\neg[H_{wt} \vee M_{wt}] \wedge \neg A_{wt}]$$

not only to

$$\lambda w \lambda t [\neg H_{wt} \wedge \neg M_{wt} \wedge \neg A_{wt}], \text{ for there would be not}$$

enough similarity, but further to

$$\lambda w \lambda t [\neg H_{wt} \wedge R_{wt} \wedge W_{wt}]$$

- this can be done (not only) in $DS^{HRW(MA)}$
- crucial claim: *verisimilitude is derivation-systems dependent*

IV.3 The Verisimilitude function (selected rows)

	$T_k:$	$T_0:$	$DS:$	$\text{ver}(T_k, T_0, DS) =$
1.	$\lambda w \lambda t [\neg H_{wt} \wedge R_{wt} \wedge W_{wt}]$	$\lambda w \lambda t [H_{wt} \wedge R_{wt} \wedge W_{wt}]$	DS^{HRW}	0,66
2.	$\lambda w \lambda t [\neg H_{wt} \wedge R_{wt} \wedge W_{wt}]$	$\lambda w \lambda t [H_{wt} \wedge R_{wt} \wedge W_{wt}]$	$DS^{\text{HRW(MA)}}$	0,66
3.	$\lambda w \lambda t [\neg H_{wt} \wedge R_{wt} \wedge W_{wt}]$	$\lambda w \lambda t [H_{wt} \wedge M_{wt} \wedge A_{wt}] *$	$DS^{\text{HRW(MA)}}$	0,66
4.	$\lambda w \lambda t [\neg H_{wt} \wedge R_{wt} \wedge W_{wt}]$	$\lambda w \lambda t [H_{wt} \wedge \neg R_{wt} \wedge W_{wt}]$	DS^{HRW}	0,33
5.	$\lambda w \lambda t [\neg H_{wt} \wedge \neg M_{wt} \wedge \neg A_{wt}]$	$\lambda w \lambda t [H_{wt} \wedge M_{wt} \wedge A_{wt}]$	DS^{HMA}	0
6.	$\lambda w \lambda t [\neg H_{wt} \wedge \neg M_{wt} \wedge \neg A_{wt}] **$	$\lambda w \lambda t [H_{wt} \wedge R_{wt} \wedge W_{wt}]$	DS^{HRW}	–
7.	$\lambda w \lambda t [\neg H_{wt} \wedge \neg M_{wt} \wedge \neg A_{wt}] *$	$\lambda w \lambda t [H_{wt} \wedge R_{wt} \wedge W_{wt}]$	$DS^{\text{HRW(MA)}}$	0,66
8.	$\lambda w \lambda t [\neg H_{wt} \wedge \neg M_{wt} \wedge \neg A_{wt}]$	$\lambda w \lambda t [H_{wt} \wedge R_{wt} \wedge W_{wt}] *$	$DS^{\text{HMA(RW)}}$	0

- * = the constructions has to be converted to equivalent construction (with the same DS); ** = the construction is not convertible
- two non-identical, equivalent and convertible constructions have the same numerical value (cf. 2. and 7. row: T_1 and T_1')
- they have distinct numerical values if they are not such (cf. 1. and 6. row)

IV.3 The derivation-systems dependence of verisimilitude

- in my (2007), I used the term “conceptual systems” (borrowed from Tichý’s follower Materna 2004 who defined a narrower notion of conceptual system)
- this was one of the reasons why Miller and Taliga reacted very negatively to my proposal (because they meant that a conceptual relativity is a very bad affair)
- note that ‘dependence’ of the verisimilitude function I propose is *technically* a very innocent one
- *philosophically*, however, it is suspicious because we have no firm intuition concerning primary/derived notions: is < primary or derived? in one DS it is primary, in another DS it is derived - there is no better answer
- the idea goes against absolutism/objectivism: no theory has a fixed *ver*

IV.4 Derivation systems and Miller's puzzle (1/2)

- I confirm Tichý's (1978) and Oddie's (1987) analysis that there are two incompatible readings of Miller's argument; in my own words (2007, 2008, 2008a):
- due to *A-reading*: T_1 and its mate T'_1 have *distinct verisimilitudes*;
- the above considerations entail that verisimilitude of T_1 and verisimilitude of T'_1 were counted in derivation systems with *distinct primary Cs*,
viz. DS^{HRW} (or perhaps $DS^{HRW(MA)}$) in the case of T_1 and the truth T_0 and, on the other hand, DS^{HMA} (or perhaps $DS^{HRW(RW)}$) for T'_1 and the truth T'_0

IV.5 Derivation systems and Miller's puzzle (2/2)

- due to *B-reading*: T_1 and is *translatable with* T'_1
- the above considerations entail that verisimilitude of T_1 and T'_1 is measured in $DS^{HRW(MA)}$ (or perhaps $DS^{HMA(RW)}$); in that case, however, T'_1 must be converted to T_1 , thus they do not count as two different theories; hence, their verisimilitude is one and the same
- to sum up: Miller's puzzle is based on *hidden equivocation of the two readings*; the source of the confusion is a tacit use (or its lack) of the 'translation rules', i.e. the definitions which make some C s derived, i.e. supervening on the primary ones

V. Conclusions

- propositions vs. conceptual content
- concluding remarks

V. Propositions vs. conceptual content

- Miller never reacted to Tichý's and Oddie's counterarguments
- in a discussions with me, Miller (esp. 2008) revealed that my (Tichý's) approach makes 'propositions' as theories, i.e. propositional constructions, *too dependent on 'conceptual' systems*; verisimilitude would then be not objective enough
- but I maintain that the remaining option is worse; let me explain:
- propositions (model structures, ...) are 'colourless' sets of possible words; that $P_1=\{W_1, W_2\}$ is closer to truth than $P_2=\{W_1, W_2, W_3\}$ is hardly of any significance
- until we know which entities are explicated by W_1 - W_3 , i.e. what is *the real conceptual content the propositions P_1 or P_2 stand for*; the conceptual content is here another word for propositional constructions

V. Concluding remarks

- I did not fully embrace the task of refutation of semantic conception of theories, adopted by Miller and many other recent philosophers of science
- I showed that an elegant and simple model of theories, according to which theories are structured meanings of some sentences, leads to useful insights in the nature of theories
- if theories are Tichý's propositional constructions, then the notion of derivation system is very relevant (though Tichý and Oddie did not notice that; verisimilitude is best defined as having derivations systems as its third parameter)
- various derivation systems are present in our conceptual scheme – one should be thus aware of this; Miller's puzzle is based on our deficiency to recognize which derivation systems

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