Entailment and Deduction in Transparent Intensional Logic

Logika: systémový rámec rozvoje oboru v ČR a koncepce logických propedeutik

pro mezioborová studia (reg. č. CZ.1.07/2.2.00/28.0216, OPVK)

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Abstract

It is sometimes objected that Tichý's logic is not logic because it underestimates deduction, provides only logical analyses of expressions. I argue that this opinion is wrong. First of all, detection of valid arguments, which are formulated in a language, needs logical analysis ascertaining which semantical entities, Tichý's so-called constructions, are involved. Entailment is defined as an extralinguistic affair relating those constructions. Validity of arguments, composed of propositional constructions, stems from properties of constructions. Such properties are displayed by derivation rules of Tichý's system of deduction.

I. Introduction

I.1 A formulation of the problem

- most recent contributions within the framework of Transparent Intensional Logic (*TIL*, founded by Pavel Tichý) belong to *logical analysis of natural language* (explication of meaning of NL expressions)
- in other words, *semantical matters* are very emphasized in TIL
- on the other hand, *deduction* is nearly entirely suppressed: it seems that no TIL-axioms, no TIL-derivation rules, briefly no TIL-axiomatic theories are stated
- since nearly every contemporary logic is presented as having something to do with deduction, one might naturally conclude that *TIL is not logic*

I.2 Some other matters related to the problem

- the distinction *logic as a language / logic as a system* (Heijenoort) does not really help here: "logic as a language" is an obsolete approach by Russell and Frege, a modern symbolic logic must be presented rather as a system (one must show completeness, prove correctness of rules, etc., which is impossible on the old "content", not formal, approach)
- the distinction *model theory (semantics) / deduction theory* does not help here as well: TIL is not put forward in terms of a deduction theory and it is not put forward in terms of a model theory either
- in a word, one cannot help but understands *TIL as being not a logic*

I.3 Basic idea behind my approach to the problem

- my solution to the problem relies on our basic idea that the *aim of logic is to determine valid arguments* (as stated in natural language)
  (*language argument* = verbal formulation of an *argument*, which is not a linguistic object)
- to check validity of a language argument, one must know *what its sentences talk about*; i.e. one must carry out a logical analysis of the language argument;
  then, a language argument is valid iff it encodes a logical structure (viz. argument) which is valid
- Tichý says even more: "knowing what we are talking about, we know what entails what" (unpublished opinion)

Content the next slides

II. Relation of logical analysis and entailment (i.e. argument validity)

 - incl. a common exposition of TIL

III. Relation of entailment and deduction

 - incl. an exposition of Tichý's system of deduction

IV. Derivation systems

V. Conclusion

II. Relation of logical analysis and entailment (i.e. argument validity)

- incl. a common exposition of TIL

II.1 Logical analysis and entailment – logical analysis of natural language

- Tichý founded TIL in the very beginning of the 1970s as a system of *intensional logic/semantics* rivalling that of Richard Montague
- Tichý soon reached very complex logical analyses of natural language expressions (belief sentences, intensional transitives, modalities, counterfactuals, temporal phenomena, verb tenses, verb aspects - episodic verbs, ...); now see esp. (Duží, Jespersen, Materna 2010)
- however, Tichý offered (already in the mid-1970s!) also *hyperintensional analyses*
- (given any class of equivalent expressions, intensional semantics treats them as having the same meaning, ignoring also their fine structure; but hyperintensional contexts are sensitive to differences of meanings thus one should not substitute expressions which are only equivalent)

II.2 Logical analysis and entailment – semantic scheme

- Tichý suggested a rather "Fregean" semantic scheme:

expression *E*

| *E expresses (in L):*

meaning = construction *C (= logical analysis of E)*

| *E denotes (in L): C constructs*:

denotatum, i.e. an intension or a non-intension

(setting aside a *referent* of *E* in *L* in a given *possible world W* and *a moment of time T*)

- *intensions* (*propositions*, *properties*,...) are functions from <*W*,*T*> couples, they have extensions (truth-values, classes of objects, ...) as their functional values

II.3 Logical analysis and entailment – objects and their constructions

- functions in *extensional* (mappings, graphs) vs. *intensional* (procedures, ...) *sense*

– constructions are structured abstract, extra-linguistic procedures ('algorithmic computations'); constructions construct objects (e.g. functions-mappings)

– *cf.* an extensive defence of the notion of construction in (Tichý 1988)

– any object O (e.g. a truth-value, proposition) is constructible by infinitely many *equivalent* (more precisely v-congruent, here v is valuation), yet *not identical*, constructions

– each construction C is *specified* by two features:

i. which object O (if any) is v-constructed by C

ii. how C constructs O (by means of which subconstructions)

– note that *constructions are closely connected with objects*

II.4 Logical analysis and entailment – constructions; semantics of TIL λ-terms

- where X is any object or construction and C or C_i is any construction (cf. Tichý 1988):

a. variables	x	(not as letters!)
b. trivializations	${}^0X$	(‘constants’)
c. compositions	$[C\ C_1 ... C_n]$	(‘applications’)
d. closures	$\lambda x C$	(‘λ-abstractions’)

- (in Tichý 1988, ramified theory of types replaces his early simple type theory)
- the TIL λ-terms are used only to *denote* constructions (*cf.* Raclavský 2010)
- the TIL λ-terms has *fixed interpretation* (one needs no tool, e.g. an axiomatic system, to supplement the TIL λ-terms with a “meaning”)

II.5 Logical analysis and entailment – example of logical analysis

- "The Pope is bald" expression *E*
- $\lambda w \lambda t\ [{}^0\mathrm{Bald}_{wt}\ {}^0\mathrm{Pope}_{wt}]$ construction *C* (= logical analysis of *E*, meaning of *E*)
- the parts of *C* are logical analyses of parts of *E* (=fine-grained logical analysis)
- the construction *C* is equivalent to (infinitely) many constructions which constructs the proposition *P*:

$$<\mathrm{W}_1,\mathrm{T}_1> \rightarrow \mathrm{T}$$ the denotatum of *E*

$$<\mathrm{W}_1,\mathrm{T}_2> \rightarrow \mathrm{F}$$

$$\vdots \qquad \vdots$$

- in hyperintensional contexts (e.g. belief attitudes, "Xenia believes that the Pope is bald"), one is related to $\lambda w \lambda t\ [{}^0\mathrm{Bald}_{wt}\ {}^0\mathrm{Pope}_{wt}]$, not to the proposition *P*; *cf.* e.g. $\lambda w \lambda t\ [{}^0\mathrm{Bald}_{wt}\ {}^0\mathrm{Xenia}\ {}^0\lambda w \lambda t\ [{}^0\mathrm{Bald}_{wt}\ {}^0\mathrm{Pope}_{wt}]]$
- let $\boldsymbol{X}$ abbreviate ${}^0X$

II.6 Logical analysis and entailment – definitions

- dependence of linguistic form on its logical form ('...*R*...in...' is a language-relative relation, '...*R**...' is a relation independent of language)

(a class of) sentences s_1, ..., s_n *entail in* language *l* a sentence *s* iff what is expressed by s_1, ..., s_n in *l* (i.e. a class of constructions) entails* what is expressed by *s* in *l* (i.e. a construction)

a language argument *a* is *valid in* language *l* iff *a*'*s* premises s_1, ..., s_n entail in *l* *a*'*s* conclusion *s*

- we need to enlighten the relation ENTAILS* (see the next slide)

II.7 Logical analysis and entailment – further definitions; conclusion

- entailment* is defined in terms of constructions and their truth:

a construction c^k is *true* in w* at t iff there exists a truth-value o such that o is a value of what (viz. a proposition) is constructed by c^k in w at t and o is identical with the truth-value T

(a class of) constructions c^k_1, ..., c^k_n *entails** a construction c^k iff for every w and t it holds that if c^k_1, ..., c^k_n, are true* in w at t, then c^k is also true* in w at t

- *conclusion*: logical analyses (i.e. constructions) of expressions are indubitably related to entailment; *if we know what sentences mean (i.e. which constructions) we are capable to determine what entails what*

- but how it relates to deduction?

II.8 Formal definitions

- let "/" abbreviates "*v*-constructs an object of type"; let o/ο (a truth-value); T/ο (the truth-value T); $p_{(i)}/ο_{\tau\omega}$ (a proposition); w/ω;t/τ; $c_{(i)}^{k}/*_k$ (a k-order construction); $\wedge,\rightarrow,=/(οοο)$ (familiar truth-functions); $\exists/(ο(οο))$; $\forall^{\omega}/(ο(ο\omega))$; $\forall^{\tau}/(ο(ο\tau))$; ${}^2C$ *v*-constructs the object, if any, *v*-constructed by C:

$[\mathbf{True}^{\pi\mathrm{T}}{}_{wt}\ p]$ $\Leftrightarrow_{\mathrm{df}}$ $\exists\lambda o[[o{=}p_{wt}]\wedge[o{=}\mathrm{T}]]$ where $\mathbf{True}^{\mathrm{T}\pi}/(οο_{\tau\omega})_{\tau\omega}$ (a property of propositions)

$[\mathbf{True}^{*k\mathrm{PT}}{}_{wt}\ c^k]$ $\Leftrightarrow_{\mathrm{df}}$ $[\mathbf{True}^{\pi\mathrm{T}}{}_{wt}\ {}^2c^k]$ where $\mathbf{True}^{\mathrm{PT}*k}/(ο*_k)_{\tau\omega}$ (a property of k-order constructions)

$\mathbf{ConjPr}\{p_1, \ldots, p_n\}$ $\Leftrightarrow_{\mathrm{df}}$ $\lambda w\lambda t[p_{1wt}\wedge \ldots \wedge p_{nwt}]$ where $\mathbf{Con}/(ο_{\tau\omega}(οο_{\tau\omega}))$ (a function from a class of propositions to propositions)

$\{p_1, \ldots, p_n\} \models^{\pi} p$ $\Leftrightarrow_{\mathrm{df}}$ $\forall^{\omega}\lambda w\forall^{\tau}\lambda t\ [\mathbf{ConjPr}\{p_1, \ldots, p_n\}_{wt}\rightarrow p_{wt}]$ where $\models^{\pi}/(οο_{\tau\omega}ο_{\tau\omega})$ (the relation $\subseteq$ between propositions)

$\{c_1{}^k, \ldots, c_n{}^k\} \models^{k} c^k$ $\Leftrightarrow_{\mathrm{df}}$ $\{{}^2c_1{}^k, \ldots, {}^2c_n{}^k\} \models^{\pi} {}^2c^k$ where $\models^{k}/(ο*_k*_k)$ (a relation between k-order constructions)

II.8 Formal definitions (cont.)

- a *language* L can be viewed as a normative system enabling speakers to communicate; such system produces (or guarantees) a family of *codes* (forming a hierarchy), i.e. functions from expressions to meanings; any code L^k can be modelled as a function from numbers (i.e. $n_{(i)}/\tau$) to k-order constructions, i.e. $L^k/(*_k\tau)$, thus also $l^k/(*_k\tau)$ (JR 2009, 2014b); *semantic notions* are modelled as related to L^k (JR 2009, 2012a, 2014a)

$$[\mathbf{EntailIn}^k\ \{n_1, ..., n_n\}\ n\ l^k] \quad \Leftrightarrow_{df} \quad \{[l^k n_1], ..., [l^k n_n]\} \models^k [l^k n] \qquad \text{where } \mathbf{EntailIn}^k/(o(o\tau)\tau(*_k\tau))$$

(a relation between classes of expressions, expressions and k-order codes)

(or: $[\mathbf{EntailIn}^k\ \{n_1, ..., n_n\}\ n\ l^k] \quad \Leftrightarrow_{df} \quad \forall^{\omega}\lambda w\forall^{\tau}\lambda w\ [\{[\mathbf{True}^{PT*k}{}_{wt}\ [l^k n_1]], ..., [\mathbf{True}^{PT*k}{}_{wt}\ [l^k n_n]]\} \rightarrow [\mathbf{True}^{PT*k}{}_{wt}\ [l^k n]]\ ]$)

- let argument be a sequence of expressions $a/(\tau\tau)$; $\mathbf{PremisesOf}/((o\tau)(\tau\tau))$; $\mathbf{ConclusionOf}/(\tau(\tau\tau))$;

$$[\mathbf{ValidIn}^k\ a\ l^k] \quad \Leftrightarrow_{df} \quad [\mathbf{EntailIn}^k\ [\mathbf{PremisesOf}\ a]\ [\mathbf{ConclusionOf}\ a]\ l^k]$$

where $\mathbf{ValidIn}^k/(o(\tau\tau))$ (a relation between sequences of expressions and codes)

III. Relation of entailment and deduction

- incl. an exposition of Tichý's system of deduction

III.1 Entailment and deduction - system of deduction

- system of deduction esp. in (Tichý 1986, 1982, which is an extract from Tichý 1976)
- only essentials are presented here
- $X{:}C$ is a *match* M which says that the simple or compound construction C constructs the/an object O (where X is the trivialization of O, a variable x for objects such as O, or nothing - empty match)
- $\Phi \Rightarrow M$ is a *sequent* where Φ contains some matches and M is a match; sequent is *valid* if every valuation satisfying members of Φ satisfy also M
- $\Phi_1 \Rightarrow M_1$; $\Phi_2 \Rightarrow M_2$; ... ; $\Phi_n \Rightarrow M_n \models \Phi \Rightarrow M$ is a *derivation rule* R, i.e. a validity preserving operations on sequents
- (*derivations* are strings of sequents where a sequent is derived according to some class of derivation rules $\vdash_R \Phi \Rightarrow M$)

III.2 Entailment and deduction - measuring validity of an argument by a sequent

- where p or $p_{(i)}$ is a variable for propositions, C^π or $C^\pi{}_{(i)}$ is a construction of a proposition, the categorical rule:

$$\models \{ p_1{:}C^\pi{}_1, p_2{:}C^\pi{}_2, \ldots, p_n{:}C^\pi{}_n\} \Rightarrow p{:}C^\pi$$

is often used to measure the validity of the argument:

$$C^\pi{}_1, C^\pi{}_2, \ldots, C^\pi{}_n \,/\, C^\pi$$

- (according to Tichý, inferences are made from arguments, not from mere sentences; *cf.* his arguments in favour of the "two-dimensional" notion of inference, 1988, 1999, Pezlar 2014)
- *realize* that the *validity of sequents depends on what objects (e.g. propositions) are constructed by constructions involved in it*; quite analogously, *validity of arguments depend on what objects (viz. propositions) are constructed by constructions involved in it*

III.3 Entailment and deduction - derivation rules exhibiting properties of objects

- to help to understand the previous slide: *derivation rules exhibit properties of* (and relations between) *objects and their constructions* (Raclavský, Kuchyňka 2011)
- for instance,

$$\Phi \cup \{\mathrm{T} : p_1\} \Rightarrow \mathrm{T} : p_2 \models \Phi \Rightarrow \mathrm{T} : [p_1 \rightarrow p_2]$$

is a rule displaying one property of implication (material conditional, constructed by $\rightarrow$), viz. that it maps $\langle \mathrm{T},\mathrm{T} \rangle$ to T

- the rule owes its validity, inter alia, to the fact that implication has that property

- *conclusion*: given Tichý's framework, entailment (which is related to logical analysis) is closely related to deduction

IV. Derivation systems

- incl. the final conclusion

IV.1 Derivation systems - the notion of derivation system

- one can still object that TIL is not a logic in a proper sense, referring here to such things such as calculi (axiomatic systems), completeness, etc.
- to resist this objection, the notion of derivation system is to be introduced
- derivation system is something like an axiomatic system in the objectual sense
- simplifying the definition from (Raclavský, Kuchyňka 2010), a *derivation system DS* is a couple:

$$DS = \langle CS, R \rangle$$

where *CS* is a class of constructions and *R* a class of derivation rules operating on *CS*
- *derivation systems DSs are tools for stating and proving facts about objects which are constructed by constructions included in CS*

IV.2 Derivation systems – classical propositional logic: area/subject matter

- let us use classical propositional logic (CPL) to illustrate our topic
- for CPL, a number of DSs can be introduced
- the *area* of each such DS consists of (total) truth-functions and two truth-values
- the *subject matter* of each such DS, however, consists of *some constructions* of the objects included in the area:
  - a) 'propositional' variables (i.e. variables for truth-values) $o_1, ..., o_n$;
  - b) trivializations of truth-values, i.e. **T** and **F**, and trivializations of truth-functions, i.e. $\neg$, $\rightarrow$, etc.
  - c) compositions made from constructions mentioned in a) and b)
- (there is a great number of constructions which construct members of the area$_{\mathrm{CPL}}$ which are not in the subject matter of any classical DS for CPL)

IV.3 Derivation systems - classical propositional logic: properties of DSs

- if DSs for CPL are introduced in the style of axiomatic systems, one enumerates selected:

  a) tautological constructions $\in CS_{DS}$ (i.e. constructing T for any valuation) as *axioms* (alternatively, axioms are certain categorical rules); e.g. $[o_1 \rightarrow [o_2 \rightarrow o_1]]$

  b) *basic derivation rules* $\in R_{DS}$, e.g. $\models \Phi \cup \{o{:}[o_1 \rightarrow o_2], o{:}o_1\} \Rightarrow o{:}o_2$ (modus ponens)

- if one can reach all tautological constructions $\in CS_{DS}$ by means of derivations which together have members of axioms$_{CSDS}$ as starting points, the DS_{CPL} in question is complete

IV.4 Derivation systems - classical propositional logic: generality of type theory

- all classical DSs for CPL are only a small fragment of DSs for CPL which are possible within the framework of Tichý's logic (Tichý's type theory)
- this fact can't be surprising: any type theory is not a single derivation system, a particular calculus ("logic"), it is rather a framework for such particular calculi
- (setting here aside non-standard semantics, most of contemporary logics are very simple DSs framed within a simple theory of types; the simplicity of the DSs lies mainly in the fact that only one kind or few kinds of variables is allowed and that λ-closures are usually excluded)
- in other words, TIL is rather an infinite class of logics

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V. Conclusion

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- Tichý's Transparent Intensional Logic is not a logic in a narrow sense
    a) of a calculus for a particular class of notions, or
    b) a class of calculi for a class of notions
- Tichý's TIL is a kind of typed λ-calculus with total and partial functions and their constructions; it is thus a very, very rich logic framework (a 'langauge')

- TIL is not mere *lingua characteristica* = a tool for formalization of natural language expressions, as it appears in some its recent presentations, it is also a powerful *calculus rationator* = a tool for rigour reasoning

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