european social fund in the czech republic
EUROPEAN UNION
MINISTRY OF EDUCATION, YOUTH AND SPORTS
OP Education for Competitiveness
INVESTMENTS IN EDUCATION DEVELOPMENT

1 Tableaux in predicate logic

Exercise 1.1: Prove that the following formulas are tautologies. Use tableau method.

a) $\Phi_1 \equiv \forall x \varphi(x) \Rightarrow \neg(\exists x \neg \varphi(x))$

b) $\Phi_2 \equiv \forall x(P(x) \Rightarrow Q(x)) \Rightarrow (\forall x P(x) \Rightarrow \forall x Q(x))$

c) $\Phi_3 \equiv \forall x(\varphi(x) \wedge \psi(x)) \Leftrightarrow (\forall x \varphi(x) \wedge \forall x \psi(x))$

d) $\Phi_4 \equiv \exists y \forall x(P(x,y) \Leftrightarrow P(x,x)) \Rightarrow \neg \forall x \exists y \forall z(P(z,y) \Leftrightarrow \neg P(z,x))$

Exercise 1.2: Prove that the formula $\forall x P(x)$ is a logical consequence of the following formulas:

$$\forall x((Q(x) \vee R(x)) \Rightarrow \neg S(x))$$
$$\forall x((R(x) \Rightarrow \neg P(x)) \Rightarrow (Q(x) \wedge S(x)))$$

Exercise 1.3: Prove the following logical consequence using the tableau method. Assume that the following three statements hold:

- There exists a dragon (denote it $D/1$).
- Dragons sleep ($S/1$) or hunt ($L/1$).
- If a dragon is hungry ($H/1$), it cannot sleep.

Conclusion: *If a dragon is hungry, it hunts.*