



## 1 Resolution in predicate logic

**Exercise 1.1:** Find all possible resolvents of the following pairs of clauses:

- a)  $C_1 = \{P(x)\}, C_2 = \{\neg P(f(x))\}$
- b)  $C_1 = \{P(x), P(y)\}, C_2 = \{\neg P(x), \neg P(y)\}$
- c)  $C_1 = \{P(x, y), P(y, z)\}, C_2 = \{\neg P(u, f(u))\}$
- d)  $C_1 = \{P(x, x), \neg R(x, f(x))\}, C_2 = \{R(x, y), Q(y, z)\}$
- e)  $C_1 = \{P(x, y), \neg P(x, x), Q(x, f(x), z)\}, C_2 = \{\neg Q(f(x), x, z), P(x, z)\}$

**Exercise 1.2:** Refute the following set of clauses

$$S = \{\{P(x), \neg Q(x, f(y)), \neg R(a)\}, \{R(x), \neg Q(x, y)\}, \{\neg P(x), \neg Q(y, z)\}, \\ \{P(x), \neg R(x)\}, \{R(f(b))\}, \{Q(x, y), \neg P(y)\}\}$$

using linear resolution, LI resolution, LD resolution and SLD resolution.

## 2 SLD-trees and resolution in Prolog

**Exercise 2.1:** Find a resolution refutation of the following program and goal in Prolog.

- |                 |         |
|-----------------|---------|
| 1. $r :- p, q.$ | 5. $t.$ |
| 2. $s :- p, q.$ | 6. $q.$ |
| 3. $v :- t, u.$ | 7. $u.$ |
| 4. $w :- v, s.$ | 8. $p.$ |

?-  $w.$

**Exercise 2.2:** Draw the SLD-tree for the following Prolog program (**Program 1**) and query. Find out, how the different order of the clauses (**Program 2**) affects the SLD-tree.

**Program 1:**

```
1. p :- q,r.      4. r :- q.
2. p :- r.       5. r.
3. q :- p.
```

?- p,q.

**Program 2:**

```
1. p :- r.      4. r.
2. p :- q,r.    5. r :- q.
3. q :- p.
```

**Exercise 2.3:** Draw the SLD-trees for the following Prolog programs and goals:

**Program 1:**

```
1. p :- a,r.    5. r :- t,a.
2. a :- b.     6. r :- s.
3. a.          7. s.
4. b :- a.
```

?- p.

**Program 2:**

```
1. p :- s,t.   5. r :- w.
2. p :- q.     6. r.
3. q.          7. s.
4. q :- r.     8. t :- w.
```

?- p.

**Exercise 2.4:** Draw the SLD-tree for the following Prolog program and goal:

```
1. p(f,g).      3. p(Z,X) :- p(X,Y), p(Y,Z).
2. p(X,X).
```

?- p(Y,f).

**Exercise 2.5:** Draw the SLD-tree for the following Prolog program and goal:

- |                                |                      |
|--------------------------------|----------------------|
| 1. $p(X,Y) :- q(X,Z), r(Z,Y).$ | 7. $s(X) :- t(X,a).$ |
| 2. $p(X,X) :- s(X).$           | 8. $s(X) :- t(X,b).$ |
| 3. $q(X,b).$                   | 9. $s(X) :- t(X,X).$ |
| 4. $q(b,a).$                   | 10. $t(a,b).$        |
| 5. $q(X,a) :- r(a,X).$         | 11. $t(b,a).$        |
| 6. $r(b,a).$                   |                      |
- ?-  $p(X,X).$

**Exercise 2.6:** Find an SLD-resolution refutation of the goal ?-  $reverse([a,b,c],X).$  assuming that the predicate `reverse/2` is defined as follows:

```
reverse(L1,L2) :- rev(L1,[],L2).
rev([H|T],A,L) :- rev(T,[H|A],L).
rev([],L,L).
```