1 Transformation into CNF

Exercise 1.1: Convert the following formulas into CNF using truth tables.

a) $(p \Leftrightarrow q) \Rightarrow (\neg p \wedge r)$

b) $(p \Rightarrow q) \Rightarrow r$

c) $p \Leftrightarrow q$

Exercise 1.2: Convert the following formulas into CNF using equivalent transformations.

a) $(p \Rightarrow q) \Leftrightarrow (p \Rightarrow r)$

b) $(p \vee q) \wedge (p \wedge r)$

c) $(p \Rightarrow q) \Rightarrow r$

d) $p \Leftrightarrow q$

2 Resolution in propositional logic

Exercise 2.1: Prove the following logical consequence using resolution:
$\neg p \vee q, \neg r \Rightarrow \neg q \models p \Rightarrow r$

3 Transformation into PNF, Skolemization

Exercise 3.1: Transform the following formulas into (conjunctive) PNF:

a) $\exists z(\forall y(\exists x P(x,y) \Rightarrow Q(y,z)) \wedge \exists y(\forall x R(x,y) \vee Q(z,y)))$

b) $\forall x R(x) \Rightarrow \forall y P(y)$

c) $\forall x \exists y P(x,y) \vee \exists x \forall z R(f(x))$

d) $\forall y \exists x R(x,y) \Leftrightarrow \forall x \forall y P(x,y)$

e) $(\forall x \exists y Q(x,y) \vee \exists x \forall y P(x,y)) \wedge \neg \exists x \exists y P(x,y)$

Exercise 3.2: Convert the following formulas into a Skolem normal form:

a) $\exists z \forall y \forall x \exists y_1 \forall x_1 ((\neg P(x,y) \vee Q(y,z)) \wedge (R(x_1,y_1) \vee Q(z,y_1)))$

b) $\exists x \forall y P(a,y,x)$

c) $\forall y \exists x P(f(x),y,x)$

d) $\exists x R(x) \Rightarrow \forall x P(x)$

e) $(\forall x \exists y Q(x,y) \vee \exists x \forall y P(x,y)) \wedge \neg \exists x \exists y P(x,y)$

4 Unification

Exercise 4.1: For the following sets of literals find the most general unifier (mgu) or explain why it does not exist. Letters a, b, c represent constants.

a) $\{P(x,f(y),z), P(g(a),f(w),u), P(v,f(b),c)\}$

b) $\{P(x,f(x)), P(f(x),x)\}$

c) $\{P(a,x), P(a,y)\}$

d) $\{P(a,x), P(b,x)\}$

e) $\{P(x), R(x)\}$

f) $\{Q(h(x,y),w), Q(h(g(v),a),f(v)), Q(h(g(v),a),f(b))\}$