european social fund in the czech republic
EUROPEAN UNION
MINISTRY OF EDUCATION, YOUTH AND SPORTS
OP Education for Competitiveness
INVESTMENTS IN EDUCATION DEVELOPMENT

1 Łukasiewicz three-valued logic ($Ł_3$)

Exercise 1.1: Write a function that expresses the valuation (truth value) of the formula $p \Leftrightarrow q$ in $Ł_3$. Write also the truth table for equivalence in $Ł_3$.
We still suppose that equivalence is a shortcut for $(p \Rightarrow q) \wedge (q \Rightarrow p)$.

Solution 1.1:
$v(p \Leftrightarrow q) = min\{min\{1, 1 - v(p) + v(q)\}, min\{1, 1 - v(q) + v(p)\}\}$
or
$v(p \Leftrightarrow q) = 1 - abs(v(p) - v(q))$

$\Leftrightarrow$	1	0.5	0
1	1	0.5	0
0.5	0.5	1	0.5
0	0	0.5	1

Exercise 1.2: Find out whether the following formulas are tautologies of $Ł_3$. Use truth tables.

a) $p \Leftrightarrow \neg\neg p$

b) $p \Rightarrow p$

c) $p \vee \neg p$

d) $(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$

e) $p \Rightarrow (q \Rightarrow p)$

Solution 1.2:

a) Formula is a tautology of $Ł_3$.

p	$\neg p$	$\neg\neg p$	$p \Leftrightarrow \neg\neg p$
1	0	1	1
0.5	0.5	0.5	1
0	1	0	1

b) Formula is a tautology of $Ł_3$.

p	$p \Rightarrow p$
1	1
0.5	1
0	1

c) Formula is not a tautology of Ł$_3$.

p	$\neg p$	$p \vee \neg p$
1	0	1
0.5	0.5	0.5
0	1	1

d) Formula is not a tautology of Ł$_3$.

p	q	$\neg p$	$p \Rightarrow q$	$\neg p \vee q$	$(p \Rightarrow q) \Leftrightarrow (\neg p \vee q)$
1	1	0	1	1	1
1	0.5	0	0.5	0.5	1
1	0	0	0	0	1
0.5	1	0.5	1	1	1
0.5	0.5	0.5	1	0.5	0.5
0.5	0	0.5	0.5	0.5	1
0	1	1	1	1	1
0	0.5	1	1	1	1
0	0	1	1	1	1

e) Formula is a tautology of Ł$_3$.

p	q	$q \Rightarrow p$	$p \Rightarrow (q \Rightarrow p)$
1	1	1	1
1	0.5	1	1
1	0	1	1
0.5	1	0.5	1
0.5	0.5	1	1
0.5	0	1	1
0	1	0	1
0	0.5	0.5	1
0	0	1	1

Exercise 1.3: How many rows (without headings) has a truth table for a formula with 3 (or n) mutually distinct truth variables?

Solution 1.3: 27 (or 3^n)

Exercise 1.4: Write a formula representing the truth function f such that $f(x, y) = 1$ iff $x = y = 0.5$, and $f(x, y) = 0$ otherwise.

How many mutually distinct binary functions are there in a three-valued logic?

Solution 1.4:
$(p \Leftrightarrow \neg p) \wedge (q \Leftrightarrow \neg q)$
$V'(3^2, 3) = 3^9 = 19683$

2 Fuzzy logic

Exercise 2.1: Let us have the following truth valuation in fuzzy logic $v(p) = 0.3, v(q) = 0.8$.

a) For which valuation of r has the formula $(p \vee \neg q) \Rightarrow r$ the truth value 0.8?

b) Write a formula F containing variables p, q such that $v(F) = 0.5$.

c) For which valuation of r has the formula $r \Rightarrow (p \Rightarrow q)$ truth value less than 1?

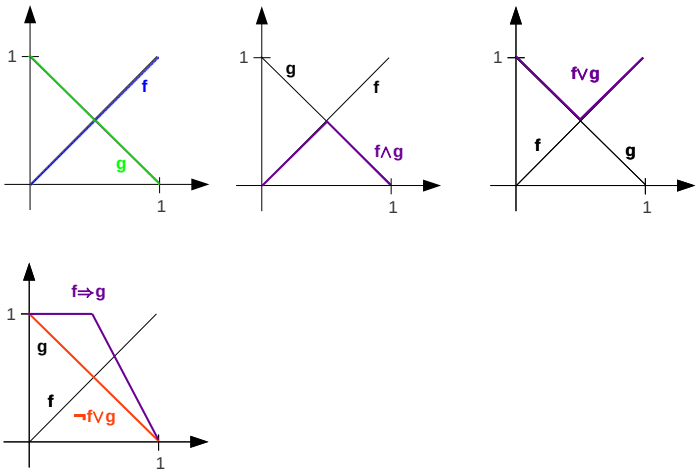
Solution 2.1:

a) $v(r) = 0.1$

b) $p \Leftrightarrow q$
$q \Rightarrow p$

c) not possible

Exercise 2.2: For the following pair of truth functions:
$v(p) = f(x) = x$ for $x \in < 0, 1 >$
$v(q) = g(x) = 1 - x$ for $x \in < 0, 1 >$
draw the graphs for conjunction and disjuncion. Then draw the graph for implication $f \Rightarrow g$ and discuss its difference from the graph of $(\neg f \vee g)$.



Solution 2.2:

Exercise 2.3: For the following truth function $v(p) = f(x)$
$f(x) = 0$, $x \in < 0, 0.2)$
$f(x) = 5x - 1$, $x \in < 0.2, 0.4)$
$f(x) = 1$, $x \in < 0.4, 1 >$
draw the graph and find an example of a phenomenon that can be represented by this function.