european social fund in the czech republic
EUROPEAN UNION
MINISTRY OF EDUCATION, YOUTH AND SPORTS
OP Education for Competitiveness
INVESTMENTS IN EDUCATION DEVELOPMENT

1 Inductive inference in propositional logic

We use Prolog notation for both the data and the learned hypotheses. For example, let us have the following data with three attributes classified into two classes `true/false`:

`Size` $\in$ {`small, medium, large`},
`Color` $\in$ {`red, blue, green`},
`Shape` $\in$ {`square, circle, triangle`}

small	red	triangle	true
small	green	triangle	true
large	red	triangle	false
small	blue	circle	false

Representation of positive examples:

```
p(small,red,triangle).
p(small,green,triangle).
```

and similarly for negative examples:

```
p(large,red,triangle).
p(small,blue,circle).
```

Our goal is to find hypotheses that cover all positive examples and don't cover any negative example. The learned hypotheses are supposed to be in the form of clauses (Prolog rules):

`p(X,Y,Z):-`
<conditions for the attributes/variables `X,Y,Z` in form `Variable=value`>

The most general hypothesis (covers all examples) is `p(X,Y,Z).`
The most specific hypothesis (covers no example) is `p(X,Y,Z):- false.`

For a clause we can get its

- minimal proper generalization by removing a condition from the body of the clause
- minimal proper specialization by adding a condition to the body of the clause

Example of a specialization:

```
p(X,Y,Z):- X=small.
```

(this is the minimal (proper) specialization of `p(X,Y,Z)`)

Exercise 1.1: For the language
`Size` $\in$ {`small, medium, large`},
`Color` $\in$ {`red, blue, green`},
`Shape` $\in$ {`square, circle, triangle`}
find a proper specialization (all proper specializations) of clauses

a) `p(Size,Color,Shape) :- Size=large, Color=red.`

b) `p(Size,Color,Shape) :- Color=red.`

c) `p(Size,Color,Shape) :- Size=large, Color=red, Shape=circle.`

d) `p(Size,Color,Shape).`

e) Find all proper specializations of the clause `p(Size,Color,Shape) :- Color=red.` that cover `p(large,red,square).`

f) Find out whether (and how) the formulas from items a)–d) are in the generalization/specialization relation.

Exercise 1.2: For the language
`0` $<$ `Size` $\leq$ `1`,
`Color` $\in$ {`red, blue`},
`Square` $\in$ {`yes, no`}
describe the space of (conjunctive) hypotheses. First, you can do this just for the language without the continuous attribute `Size`.
Then find some proper specializations of clauses

a) `p(Size,Color,Square) :- Square=yes, Color=red.`

b) `p(Size,Color,Square) :- Color=red.`

c) `p(Size,Color,Square) :- Size < 0.1, Color=red, Square=no.`

d) `p(Size,Color,Square).`

Exercise 1.3: For the following data find all its disjunctive concepts:

small	red	triangle	true
small	green	triangle	true
large	red	triangle	true
small	blue	circle	false
small	blue	triangle	false