european social fund in the czech republic
EUROPEAN UNION
MINISTRY OF EDUCATION, YOUTH AND SPORTS
OP Education for Competitiveness
INVESTMENTS IN EDUCATION DEVELOPMENT

1 Description logic

Exercise 1.1: In description logics $\mathcal{AL}$ and $\mathcal{ALCN}$ with concepts `Male`, `Female` and a role `hasChild` define the following concepts

a) `Person`

b) `Mother`, `Father`

c) `Parent`

d) `Childless`

e) `Grandmother`, `Grandfather`

f) `ParentOfSons` (a parent with at least one son)

g) `ParentOfOnlySons`

h) `MotherWithManyChildren` (a mother with more then three children)

i) `GrandmotherOfOnlyGrandsons`

Solution 1.1:

a) $\texttt{Person} \equiv \texttt{Male} \sqcup \texttt{Female}$
(assumption: there are only people in `Male` and `Female`)

$\mathcal{AL}: \texttt{Person} \equiv \top$
(assumption: there are only people in the domain)

b) $\texttt{Mother} \equiv \texttt{Female} \sqcap \exists \texttt{hasChild.Person}$
$\texttt{Father} \equiv \texttt{Male} \sqcap \exists \texttt{hasChild.Person}$
$\mathcal{AL}: \texttt{Mother} \equiv \texttt{Female} \sqcap \exists \texttt{hasChild}.\top, \texttt{Father} \equiv \texttt{Male} \sqcap \exists \texttt{hasChild}.\top$

c) $\texttt{Parent} \equiv \texttt{Person} \sqcap \exists \texttt{hasChild.Person}$
or (because we suppose that there are only people in `Person`)
$\texttt{Parent} \equiv \exists \texttt{hasChild.Person}$
$\mathcal{AL}: \texttt{Parent} \equiv \exists \texttt{hasChild}.\top$

d) $\texttt{Childless} \equiv \texttt{Person} \sqcap \neg(\exists \texttt{hasChild.Person})$
$\mathcal{AL}: \texttt{Childless} \equiv \texttt{Person} \sqcap \forall \texttt{hasChild}.\bot$

e) $\texttt{Grandmother} \equiv \texttt{Mother} \sqcap \exists \texttt{hasChild.Parent}$
$\texttt{Grandfather} \equiv \texttt{Father} \sqcap \exists \texttt{hasChild.Parent}$
$\mathcal{AL}:$ it is not possible

f) $\texttt{ParentOfSons} \equiv \texttt{Parent} \sqcap \exists \texttt{hasChild.Male}$

g) $\texttt{ParentOfOnlySons} \equiv \texttt{Parent} \sqcap \forall \texttt{hasChild}.\texttt{Male}$

h) $\texttt{MotherWithManyChildren} \equiv \texttt{Mother} \sqcap \geq 4\, \texttt{hasChild}$

i) $\texttt{GrandmotherOfOnlyGrandsons} \equiv$
$\texttt{Grandmother} \sqcap \forall \texttt{hasChild}.(\texttt{ParentOfOnlySons} \sqcup \texttt{Childless})$

Exercise 1.2: In description logic $\mathcal{ALC}$ with concepts `Male, Doctor, Rich, Famous` and roles `hasChild, hasFriend` define a popular textbook's concept `HappyFather`: "a father whose all children are doctors and all of the children have rich or famous friends".

Solution 1.2: $\texttt{HappyFather} \equiv$
$\texttt{Male} \sqcap (\exists \texttt{hasChild}.\top) \sqcap \forall \texttt{hasChild}.(\texttt{Doctor} \sqcap \exists \texttt{hasFriend}.(\texttt{Rich} \sqcup \texttt{Famous}))$

Exercise 1.3: Prove or reject the following statements using tableaux in $\mathcal{ALC}$ description logic.

a) $(\texttt{Person} \sqcap (\forall \texttt{hasChild}.\texttt{Male})) \sqsubseteq (\texttt{Person} \sqcap (\exists \texttt{hasChild}.\texttt{Male}))$

b) $(\texttt{Male} \sqcap (\exists \texttt{hasChild}.\texttt{Male}) \sqcap (\forall \texttt{hasChild}.\texttt{Male})) \sqsubseteq$
$((\texttt{Male} \sqcup \texttt{Female}) \sqcap (\exists \texttt{hasChild}.(\texttt{Male} \sqcup \texttt{Female})))$

Solution 1.3: Statements of the form $\texttt{C} \sqsubseteq \texttt{D}$ are proved using equivalent unsatisfiability of $\texttt{C} \sqcap \neg\texttt{D}$. We suppose that $\texttt{C} \sqcap \neg\texttt{D}$ is satisfiable, so it contains at least one element. The root of the constructed tableau is then $(\texttt{C} \sqcap \neg\texttt{D})(\texttt{a})$ transformed into the negation normal form. To prove the original statement we have to create a finished contradictory tableau. Ordinary tableaux are used, not signed ones (every node is supposed to be true).
The first statement should be rejected, the second one should be proved.